Job Creation, Job Destruction and the Life Cycle

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Abstract

This paper originally incorporates life-cycle features into the job creation - job destruction framework. Once a finite horizon is introduced, this workhorse labor market model naturally delivers the empirically uncontroversial prediction that the employment rate of workers decreases with age due to lower hirings and higher firings of older workers. This age profile of hirings and firings is in addition found to be optimal in a competitive search equilibrium context. If search externalities are not internalized and unemployment benefits distort equilibrium, there is a room for labor market policy differentiated by age. This lastly allows us to debate the incidence of labor demand policies which have been introduced in many countries to favor the older worker employment. We show that hiring subsidies and firing costs should be decreasing with age when unemployment benefits are sufficiently high, as in the Europe. On the contrary, if unemployment benefits are low, as in the US, optimal hiring subsidies and firing taxes should be increasing with age. In this latter case, the introduction of anti-discrimination laws is a good proxy of this first best policy.

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1 Introduction

The employment rate of older workers exhibits a large drop in major OECD countries. Considering the 55-64 range, table 1 documents this uncontroversial fact: the employment rate of older workers is cut down by 15 points in average. Both labor supply and labor demand factors has been put forward to explain these features. Retirement programs and the implicit tax on continued activity they impose have been extensively examined across OECD countries (see Gruber and Wise [1999]). They constitute a primary candidate for explaining the low older workers employment rate. The labor demand for older workers has also been scrutinized and its relative weakness certainly helps explain the decrease in the employment rate at the end of the working life. Relative marginal products and relative wages for various groups of workers are then studied in order to explain the intrinsic difficulties of older workers in the labor market (see for instance Hellerstein, Neumark and Troske [1999] and Crépon, Deniau and Perrez-Duarte [2002]). Both explanations are undoubtedly relevant to grasp the specificities of older workers. They are eligible to social security programs and suffers from the ongoing technological progress. But something is missing in this whole picture.

Table 1: Employment rate by age groups in OECD countries

	25-49	50-64	50-54	55-59	60-64
Japan	78.13	68.34	78.95	72.54	50.66
US	79.14	66.82	77.02	68.38	48.86
GB	81.37	63.86	78.53	67.45	40.02
Canada	81.21	62.91	77.49	63.38	39.3
Belgium	78.08	42.48	65.47	39.45	13.79
France	79.93	52.75	75.24	54.15	13.23
Italy	71.78	42.98	65.03	41.12	19.83
Netherlands	83.37	55.42	75.02	58.71	22.97

Let us consider a finer description of the employment rate by distinguishing workers aged 55-59 and those aged 60-64. If the profile of employment rates is clearly decreasing with age for any countries, the speed of this decrease markedly differs across countries. Two country groups emerges very clearly: those with still high employment rates for workers aged 55-59

(Canada, Great Britain, Japan and the United States) and those which already experience a huge decrease (around 25 points) at these ages (Belgium, France, Italy and the Netherlands).[2]

How can we explain this result? Should we invoke productivity, technological bias or labor cost differentials? There are no serious reasons to believe that the 55-59 year old workers in the latter countries are more particularly sensitive to these factors. Actually the decrease in the employment rate of older workers is as much important as the retirement age gets closer. As documented by Gruber and Wise [1999], the second group of countries is indeed characterized by an effective retirement age of 60 (versus 65 in the first group).

In this paper, we show that the workhorse labor market model of Mortensen and Pissarides [1994] naturally delivers such a prediction for the older workers employment rate once a finite horizon, typically determined by retirement, is explicitly taken into account.[3] As unemployment spell and job spell durations are derived from forward-looking job creation and job destruction decisions, employment should highly differ by age when ageing means a closer exit from the labor force.

We put emphasis on labor demand side which is traditionally taken into consideration only when productivity differentials exists. We aim at convincing that labor demand for older workers is crucially affected by retirement age. Moreover, it will allow us to shed light on labor demand policies before early retirement age, which has been introduced, in lot of countries, to favor the older worker employment. In France and in Finland, firing costs for older workers has been put in place to discourage firms to lay off them. In Great Britain and in France, hiring subsidies aim at favoring the exit of older workers from the unemployment.

Surprisingly enough, the extensively-used approach initiated by Mortensen and Pissarides [1994] has been very rarely integrated into a life cycle framework[4]. We propose in this paper to show that this approach constitute a natural starting point of any analysis which aims at unveiling the specificities of older workers. Both hiring and firing policies are detrimental to aged

[2]The difference from the group aged 50-54 is only of ten points on average for the former, whereas it increases by up to 25 points for the latter.

[3]As the terminal date (retirement) in the labor market is under the workers decisions, labor supply in case of costly search for unemployed people is also a natural candidate for understanding interactions between retirement and employment at the end of the working life (see HAIR/LANG/SOPR/06 who propose quantitative arguments in favor of this view.)

[4]A noticeable exception is Bettendorf and Broer [2003]. Seater [1977] also allows for life cycle feature but in a job search framework.

workers when labor markets are characterized by search and recruiting costs: a reduced working life horizon deters firms to search or hoard them. Pure rationality pushes firms to "discriminate" against older workers. In this sense, there is no discrimination against aged workers: these latter are objectively less profitable. Adding search for unemployment workers reinforces mechanisms at work in the life cycle setting: as retirement gets closer, individuals search less as the expected job duration is lower. A canonical matching model plugged into a life-cycle frame reveals that it is in the interest of firms to adopt differentiated hiring and firing policies across worker ages, because workers differ in terms of expected distance from retirement and henceforth in terms of expected job duration. There exists some fundamental forces against the "last in, first out" or age discrimination legislations which has been sometimes implemented to promote older workers employment. By the way, it helps explain their enforcement difficulties in front of a classic not-incentive-compatible problem.

Before engineering any policy devices to circumvent firms (and workers) discriminating hiring and firing behaviors, it is necessary to study their social optimality. When search is costly, minimizing rotations implies that the first best coincides with the fact that older workers, due to their impending retirement, come first (last) in the firing (hiring) process. Without any other distortions than the matching process, it is optimal that firms discriminate in their hiring and firing policies across ages, and only because of age. We then show that the decentralized equilibrium coincides even with the first best outcome either when the Hosios condition holds (with wage bargaining) or when search equilibrium is competitive (as studied by Moen [1997]).

But when search externalities are not internalized and unemployment benefits distort equilibrium there is a room for labor policies differentiated by age. As a preliminary step, we point out that the policies patterns by age affect the firings and hirings decisions: in particular, the firing decisions are highly sensitive to the firing cost profile by age because age differentiation actually introduces temporal costs variations for a given employed workers.

How should be the optimal profile by age of firing cost and hiring subsidies? Intuitively, age should also matter. We show that age constitutes the cornerstone of any optimal labor market policies, and often in an opposite way it is put in place by effective legislations. If the laisser-faire equilibrium is no more socially optimal, the firing costs and hiring subsidies policies should be shaped according to age. We show that hiring subsidies and firing costs should be decreasing with age when unemployment benefits are sufficiently high, as in the Europe. In this case, we argue that anti-discrimination legislations appears counter-productive as they benefit to older workers. On the contrary, if unemployment benefits are low, as in the US, optimal hiring subsidies and firing taxes should be increasing with age. In this latter case, the introduction of anti-discrimination laws is a good proxy of this first best policy.

The first section presents hiring and firing firms decisions when workers differs by age and the labor market . A second section is devoted to establish the first best employment age profile and at which condition decentralized decisions are optimal. The next section determines the age profile of optimal policies when the equilibrium outcome is no more an optimum. Finally, in the last section, age anti-discrimination policies are evaluated as proxies of optimal policies.

2 How Does Life Cycle Setting Affect Employment? A Job Creation - Job Destruction Approach

Let us consider an economy *à la* Mortensen - Pissarides [1994], *i.e.* a labor market with frictions: there is a costly delay in the process of filling vacancies. Unemployed workers search effort is discarded for matter of simplicity. We present in appendix a version with endogenous search for unemployed people to verify it does not alter the main conclusions. Job destructions are endogenous and deeply interplay with job creations. Wages are determined by a specific sharing rule of the rent generated by a job that can be interpreted as the result of a bargaining between workers and employers. At this stage, no other frictions or inefficiencies are introduced.

Contrary to the large literature following Mortensen - Pissarides [1994], we consider a life cycle setting characterized by a deterministic age at which workers exit the labor market. Firms are free to target their hirings by age: directed-search by age is technologically possible and legally authorized. Consistently, we are considering at this stage a "*laissez-faire*" economy, since anti-discrimination laws in use in most of OECD countries actually rely on labor market policy tools.

2.1 Benchmark Model Description

2.1.1 Workers Flows

We consider a discrete time model and assume that at each period, the older workers generation leaving the labor market is replaced by a younger workers generation of the same size (normalized to unity) so that there is no labor force growth in the economy. We denote i the worker's age and T the exogenous age at which workers exit the labor market. There is no heterogeneity across workers and this age is perfectly known by employers. We assume that each workers of the new generation enters into the labor market as unemployed.

Job creation takes place when a firm and a worker meet. Firms are small and each has one job. The flows of newly created jobs result from a matching function which inputs are vacancies and unemployed workers. The destruction flows derive from idiosyncratic productivity shocks that hit randomly the jobs. Once a shock arrives, the firm has no choice but either to continue production or to destroy the job. Then, for age $i \in (2, T-1)$, employed workers are faced to layoffs when their job become unprofitable. At the beginning of each period, a job productivity ϵ is drawn in the general distribution $G(\epsilon)$ with support in the $[0, \bar{\epsilon}]$. Firms decide to close down any jobs which productivity is below a (endogenous) productivity threshold (productivity reservation) denoted R_i.

Let u_i be the unemployment rate and v_i the vacancy rate of age i. For any age, we assume that there is matching functions that give the number of jobs as a function of the number of vacancies and the number of unemployed workers $M(v_i, u_i)$ where M is increasing in both its arguments, concave and CRS. Let $\theta_i = v_i/u_i$ denote the tightness of the labor market of age i. It is then straightforward to define the probability of filling a vacancy as $q(\theta_i) \equiv \frac{M(u_i,v_i)}{v_i}$ and the probability for unemployed workers to meet a vacancy as $p(\theta_i) \equiv \frac{M(u_i,v_i)}{u_i}$.

At the beginning of their age i, the realization of the productivity level on each job is revealed. Workers hired when they were $i-1$ years old (at the end of the period) are now productive. Workers which productivity is below the reservation productivity R_i are laid off. For any age i, the flow from employment to unemployment is then equal to $G(R_i)(1-u_{i-1})$. The other workers who remain employed $(1-G(R_i))(1-u_{i-1})$ can renegociate their wage.

The dynamics by age of unemployment is then given by:

$$u_i = u_{i-1}\left(1 - p(\theta_{i-1})\right) + G(R_i)(1-u_{i-1}) \quad \forall i \in (2, T-1) \tag{1}$$

for a given initial condition $u_1 = 1$. The overall unemployment rate u is then defined by $u = \frac{\sum_{i=1}^{T-1} u_i}{T-1}$.

2.1.2 Firms' Hiring Decision

Any firms is free to open a job vacancy and engage in hiring. c denotes the flow cost of recruiting a worker and $\beta \in [0,1]$ the discount factor. Let V_i be the expected value of a vacant job directed to a worker of age i:

$$V_i = -c + \beta \left[q(\theta_i) J_{i+1}(\bar{\epsilon}) + (1 - q(\theta_i)) V_i \right]$$

where $J_i(\epsilon)$ is the expected value of a filled job by a worker of age i with idiosyncratic productivity ϵ. Following Mortensen and Pissarides, we assume that new jobs start at the highest productivity level, $\epsilon = \bar{\epsilon}$.

As $J_T(\bar{\epsilon}) = 0$, no firms search workers of age $T-1$: $\theta_{T-1} = 0$. The zero-profit condition $V_i = 0, \quad \forall i \in (1, T-2)$ allows us to determine the vacancy rate v_i and the labor market tightness θ_i:

$$\beta J_{i+1}(\bar{\epsilon}) = \frac{c}{q(\theta_i)} \tag{2}$$

As $1/q(\theta_i)$ is the expected duration of a vacancy directed to a worker of age i, the market tightness is such that the expected and discounted job value is equal to the expected cost of hiring a worker of age i.

2.1.3 Firms' Firing Decision

For a bargained wage $w_i(\epsilon)$, the expected value $J_i(\epsilon)$ of a filled job by a worker of age i is defined by:

$$J_i(\epsilon) = \epsilon - w_i(\epsilon) + \beta \int_{R_{i+1}}^{\bar{\epsilon}} J_{i+1}(x) dG(x) + \beta G(R_{i+1}) \max_i \{V_i\} \quad \forall i \in [1, T-1] \tag{3}$$

It is worth emphasizing that the deterministic exit at age T leads to an exogenous job destruction, whatever the productivity realization: $J_T(\epsilon) = 0$.

The (endogenous) job destruction rule[5] $J_i(\epsilon) < 0$ leads to a reservation productivity R_i defined by $J_i(R_i) = 0, \quad \forall i \in [2, T-1]$:

$$R_i = w_i(R_i) - \beta \int_{R_{i+1}}^{\bar{\epsilon}} J_{i+1}(x) dG(x) - \beta G(R_{i+1}) \max_i \{V_i\} \quad \forall i \in [2, T-1] \tag{4}$$

The higher the wage, the higher the reservation productivity, and hence the job destruction flows. On the opposite, the higher the option value of

[5] Under bargaining wage, this destruction is also in the interest of the worker.

occupied jobs (expected gains in the future), the weaker the job destructions. Because the job value vanishes at the end of the working life, labor hoarding of older workers is less profitable. It is again worth determining the terminal age condition: $R_{T-1} = w_{T-1}(R_{T-1})$.

2.1.4 The Wage Bargaining

The rent to a job is divided between the employer and the worker by the wage rule. Following the most common specification, wages are determined by the Nash solution to a bargaining problem.

It remains to determine the values of employed (on a job of productivity ϵ) and unemployed workers of any age i, $\forall i < T$. They are respectively given by:

$$\mathcal{W}_i(\epsilon) = w_i(\epsilon) + \beta \left[\int_{R_{i+1}}^{\bar{\epsilon}} \mathcal{W}_{i+1}(x) dG(x) + G(R_i)\mathcal{U}_{i+1} \right] \tag{5}$$

$$\mathcal{U}_i = b + \beta \left[p(\theta_i)\mathcal{W}_{i+1}(\bar{\epsilon}) + (1 - p(\theta_i))\mathcal{U}_{i+1} \right] \tag{6}$$

with $b \geq 0$ the opportunity cost of employment.[6]

For a given bargaining power of the workers, considered as constant across ages, the global surplus generated by a job, $S_i \equiv J_i(\epsilon) + W_i(\epsilon) - U_i$, is divided according to the following sharing rule:

$$\mathcal{W}_i(\epsilon) - \mathcal{U}_i = \gamma \left[J_i(\epsilon) + \mathcal{W}_i(\epsilon) - \mathcal{U}_i \right] \tag{7}$$

It is shown in Appendix how to obtain the following expression for the bargained wage:

$$w_i(\epsilon) = (1 - \gamma)b + \gamma (\epsilon + c\theta_i) \quad \forall i \in [1, T-1] \tag{8}$$

This is a traditional wage equation, except that age matters through the market tightness. As this latter diminishes along the life cycle, the age profile of wage is decreasing. This could counteract the incentives for firms to fire old workers.

2.2 The "*Laissez-Faire*" Equilibrium

The main objective of this section is to characterize the life cycle pattern of hirings and firings. For didactic reasons, we first relies exclusively on the firm behavior, without considering wages retroactions. Wages are assumed

[6] We assume that $\mathcal{W}_T = \mathcal{U}_T$ so that the social security provisions do not affect the wage bargaining and the labor market equilibrium.

to be fixed at the reservation wage level b. This "wage posting" case could be rationalized by a bargaining power for workers equal to 0 ($\gamma = 0$ in (8)). Then, we will turn to our benchmark labor market equilibrium when it is allowed for wages adjustments over the life cycle. The introduction of endogenous search effort and its implications on equilibrium job creations and job destructions are then examined. Lastly, we turn to the age profile of employment rates and discuss its properties in our benchmark equilibrium.

2.2.1 The Partial Equilibrium: A Wage Posting Case

If wages are equal to b, the firing policy, defined by R_i, is independent to the hiring one.

Proposition 1. *If $\gamma = 0$, a labor market equilibrium with wage posting exists and it is characterized by $\{R_i, \theta_i\}$ solving:*[7]

$$\begin{array}{lll} \frac{c}{q(\theta_i)} & = & \beta(\bar{\epsilon} - R_{i+1}) \qquad\qquad\qquad (JC_{PartialEq}) \\ R_i & = & b - \beta \int_{R_{i+1}}^{\bar{\epsilon}} [1 - G(x)]dx \quad (JD_{PartialEq}) \end{array}$$

with terminal conditions $R_{T-1} = b$ and $\theta_{T-1} = 0$.

Proof. See appendix B.1. □

It is then possible to derive the age profile of hirings and firings along the life cycle.

Property 1. *$R_{i+1} \geq R_i$ and $\theta_{i+1} \leq \theta_i \quad \forall i$.*

Proof. See appendix B.2. □

Older workers are more fragile faced to idiosyncratic shocks. A shortened horizon relative to younger workers make them more exposed to firings. Otherwise stated, this reflects that labor hoarding decreases with worker's age. In turn, it creates a downward pressure on the hirings of older workers.

2.2.2 The Benchmark Equilibrium with Wage Bargaining

If wages are bargained according to the equation (8), the firing policy depends now on the market tightness. As presented above, this effect could put into question the decreasing age profile of firings since wages could compensate for the horizon shortening effect. Furthermore, the relationship between the reservation productivity and the age could be reversed.

[7] The system of forward variables (equations ($JC_{PartialEq}$)-($JD_{PartialEq}$)), can be solved independently to unemployment dynamics.

Proposition 2. *A labor market equilibrium with wage bargaining exists and it is characterized by* $\{R_i, \theta_i\}$ *solving:*

$$\begin{array}{rcll} \frac{c}{q(\theta_i)} & = & \beta(1-\gamma)(\bar{\epsilon}-R_{i+1}) & (JC) \\ R_i & = & b+\left(\frac{\gamma}{1-\gamma}\right)c\theta_i-\beta\int_{R_{i+1}}^{\bar{\epsilon}}[1-G(x)]dx & (JD) \end{array}$$

with terminal conditions $R_{T-1}=b$ *and* $\theta_{T-1}=0$.

Proof. See Appendix B.3. □

Corollary 1. *Let be* $M(v,u)=v^{\psi}u^{1-\psi}$ *with* $0<\psi<1$, *and* $G(\epsilon)=\frac{\epsilon}{\bar{\epsilon}}$, $\forall\epsilon\in[0,\bar{\epsilon}]$, *with* $b\leq\bar{\epsilon}\leq 2b/\beta$,[8] *the labor market equilibrium with wage bargaining can be summarized by a sequence* $\{R_i\}_{i=2}^{T-1}$ *solving:*

$$R_i=b+\left(\frac{\gamma c}{1-\gamma}\right)\left[\frac{\beta(1-\gamma)}{c}(\bar{\epsilon}-R_{i+1})\right]^{\frac{1}{1-\psi}}-\frac{\beta}{2\bar{\epsilon}}(\bar{\epsilon}-R_{i+1})^2 \quad (9)$$

with terminal condition $R_{T-1}=b$.

Proof. Straightforward. □

The sequence of R_i is no more necessarily monotone. If the wage deceases sufficiently at the end of working life because of the weakness of the market tightness, then firms could fire first the younger workers. The following property and corollary state restrictions implying that this indirect effect of age through wages does not dominate the direct impact of age on labor hoarding and firing.

Property 2.

$$if\ 1\geq\begin{cases}\frac{\gamma}{1-\psi}\left[\frac{\beta(1-\gamma)\bar{\epsilon}}{c}\right]^{\frac{\psi}{1-\psi}} & for\ \psi\geq 1/2\\ 2\gamma\bar{\epsilon}\left[\frac{\beta(1-\gamma)}{c}\right]^{\frac{\psi}{1-\psi}}(\bar{\epsilon}-b)^{\frac{2\psi-1}{1-\psi}} & for\ \psi\leq 1/2\end{cases}$$

then the labor market equilibrium verifies $R_{i+1}\geq R_i$ *and* $\theta_{i+1}\leq\theta_i$ $\forall i$.

Proof. See Appendix. □

[8] From (9) it is straightforward to see that $b\leq\bar{\epsilon}\leq 2b/\beta$ is sufficient for an interior solution to exist ($R_i\geq 0\ \forall i$).

It is worth noting that, for $\gamma \to 0$, the equilibrium is characterized by $R_{i+1} \geq R_i$, whatever the values taken by the structural parameters. Otherwise the value c of the recruiting costs is central for understanding this result. It determines how the age influences the vacancy rate. The higher the recruiting cost, the steeper the age profile of wages. If c is sufficiently high, the wage effect cannot counteract the horizon effect on the reservation productivity: the age profile of firing are still decreasing.

Corollary 2. *If $\psi = 1/2$ the condition $c \geq \beta\gamma(1-\gamma)2$ ensures that the labor market equilibrium verifies $R_{i+1} \geq R_i$ and $\theta_{i+1} \leq \theta_i \;\; \forall i$.*

Proof. Straightforward from property 2 with $\psi = 1/2$. □

2.2.3 The Role of Endogenous Search Effort

For didactic reasons, until now we neglect the incidence of life cycle on workers' search effort. Making the latter endogenous would actually reinforce the decrease in the employment rate at the end of working life. As the retirement age gets closer, the return of search investments decreases because of the horizon (the expected job duration) over which they can recoup their investment is reduced. This point can easily be stated by considering the following unemployed problem to define search intensity (see the appendix A for a detailed description of the model with a more general specification of preferences):

$$\mathcal{U}_i = \max_{e_i} \left\{ b - \frac{e_i^2}{2} + \beta \left[e_i p(\theta_i) \mathcal{W}_{i+1}(1) + (1 - e_i p(\theta_i)) \mathcal{U}_{i+1} \right] \right\}$$

where the labor market tightness is now defined by $\theta_i \equiv v_i / [\overline{e_i} u_i)]$ with a matching function $M(v_i, \overline{e}_i u_i)$.

The optimal decision rule shows that it is older unemployed workers' interest to reduce their search intensity since the discounted sum of surplus related to employment is decreasing with age:

$$e_i = \beta p(\theta_i) \left[\mathcal{W}_{i+1}(1) - \mathcal{U}_{i+1} \right]$$

This provides an additional source to existing forces that lead job creation to decrease with age. This point can be highlighted by examining the equilibrium properties with search effort.

Proposition 3. *A labor market equilibrium with wage bargaining and endogenous search effort exists and it is characterized by $\{R_i, \theta_i\}$ solving:*

$$\begin{aligned} \tfrac{c}{q(\theta_i)} &= \beta(1-\gamma)(\overline{\epsilon} - R_{i+1}) && (JC) \\ R_i &= b + \tfrac{1}{2}\left(\tfrac{\gamma}{1-\gamma} c\theta_i\right)^2 - \beta \textstyle\int_{R_{i+1}}^{1} [1 - G(x)]dx && (JD_{eff}) \end{aligned}$$

with terminal conditions $R_{T-1} = b$ and $\theta_{T-1} = 0$.

Proof. See Appendix. □

Corollary 3. *If $\psi = 1/2$ the condition $c \geq \beta\gamma(1-\gamma)\sqrt{2}$ ensures that the labor market equilibrium verifies $R_{i+1} \geq R_i$ and $\theta_{i+1} \leq \theta_i \;\; \forall i$.*

This clearly shows that the restriction on c to see the equilibrium with $R_{i+1} \geq R_i$ emerging is weaker than imposed by corollary 2. This point demonstrates the positive role of search effort in explaining the increase (decrease) of firings (hirings) with age.

2.2.4 The Age Profile of Employment Rate

The age profile of hirings and firings has been recursively determined from terminal conditions. On the contrary, the age profile of unemployment u_i (or employment $n_i = 1 - u_i$) depends on an arbitrary initial condition u_1. This explains why it is ambiguous:

$$u_i \gtrless \frac{G(R_{i+1})}{G(R_{i+1}) + p(\theta_i)} \quad \Rightarrow \quad n_{i+1} \gtrless n_i \;\; \forall i$$

Let us denote $\Psi(R_{i+1}, \theta_i) = \frac{G(R_{i+1})}{G(R_{i+1})+p(\theta_i)}$. By definition, $\frac{\partial \Psi(R_{i+1},\theta_i)}{\partial R_{i+1}} > 0$ and $\frac{\partial \Psi(R_{i+1},\theta_i)}{\partial \theta_i} < 0$. For $\theta_{i+1} \leq \theta_i$ and $R_{i+1} \geq R_i$ from the property 2, it appears that $\Psi(R_{i+1}, \theta_i) \leq \Psi(R_{i+2}, \theta_{i+1}) \;\; \forall i$.

Property 3. *Let consider that $\{R_2, \theta_1\}$ verifies proposition 2, if $u_1 > \Psi(R_2, \theta_1)$, there exists a threshold age $\tilde{T}$ so that $n_i \geq n_{i-1} \;\; \forall i \leq \tilde{T}$ and $n_i \leq n_{i-1} \;\; \forall i \geq \tilde{T}$.*

Proof. See Appendix. □

Corollary 4. *If $u_1 = 1$, there exists a threshold age $\tilde{T}$ so that $n_i \geq n_{i-1} \;\; \forall i \leq \tilde{T}$ and $n_i \leq n_{i-1} \;\; \forall i \geq \tilde{T}$.*

Proof. The proof is straightforward since $\Psi(R_2, \theta_1) < 1$. □

In the case where all the new entrants are unemployed, high vacancy rates and weak firing rates at the beginning of the working life cycle make the employment rate increasing with age until the age $\tilde{T}$. From $\tilde{T}$ on, the employment rate evolution by age mimics the age profile of firings and hirings. The age heterogeneity across workers in the context of a life cycle leads to low employment rate for older workers.

2.3 Efficient Job Creation and Job Destruction Rates over the Life Cycle

This paper showed that it is firms' interest to hire (fire) less (more) older workers than younger ones. A *laissez-faire* equilibrium is then typically featured by job creation (destruction) rates decreasing (increasing) with age.

This section wonders to what extent these labor market equilibrium outcomes are optimal. We precisely show that either Hosios condition or competitive search equilibrium *à la* Moen [1997] lead to equilibirum efficiency. But, in general (if markets are incomplete), efficiency is not achieved.

2.3.1 The Hosios Condition Revisited

In line with the analysis of Pissarides [2000] with infinite lived agents, we derive the optimal steady-state allocation by maximizing the sum of discounted output flows net of recruiting costs. This is done over the life cycle of workers. We will show hereafter that it is equivalent to maximize the expected gain of unemployed workers. Currently, the efficient problem is stated as:

$$\max_{\{R_i\}_{i=2}^{T-1},\{\theta_i\}_{i=1}^{T-1}} \sum_{i=1}^{T-1} \beta^i \left(y_i + bu_i - c\theta_i u_i\right) \tag{10}$$

under the constraints:

$$u_{i+1} = G(R_{i+1})(1-u_i) + u_i\left(1-p(\theta_i)\right) \tag{11}$$

$$y_{i+1} = u_i p(\theta_i) + (1-u_i)\int_{R_{i+1}}^{\overline{\epsilon}} \epsilon dG(\epsilon) \tag{12}$$

where y_i is the average output.

Proposition 4. *Let $\eta(\theta_i) = -\theta_i q'(\theta_i)/q(\theta_i)$, the efficient allocation exists and it is characterized by $\{R_i^\star, \theta_i^\star\}$ solving:*

$$\frac{c}{q(\theta_i^\star)} = \beta\left(1-\eta(\theta_i^\star)\right)\left(\overline{\epsilon} - R_{i+1}^\star\right) \qquad (JC^\star)$$

$$R_i^\star = b + \frac{\eta(\theta_i^\star)}{1-\eta(\theta_i^\star)} c\theta_i^\star - \beta\int_{R_{i+1}^\star}^{\overline{\epsilon}} \left[1-G(x)\right]dx \qquad (JD^\star)$$

with terminal conditions $R_{T-1}^\star = b$ and $\theta_{T-1}^\star = 0$.

Proof. See Appendix. □

Property 4. *Let $\eta(\theta_i^\star) = 1-\psi \quad \forall i$,*

$$if\ 1 \geq \begin{cases} \left(\frac{\beta\psi\overline{\epsilon}}{c}\right)^{\frac{\psi}{1-\psi}} & for\ \psi \geq 1/2 \\ 2(1-\psi)\overline{\epsilon}\left(\frac{\beta\psi}{c}\right)^{\frac{\psi}{1-\psi}}\left(\overline{\epsilon}-b\right)^{\frac{2\psi-1}{1-\psi}} & for\ \psi \leq 1/2 \end{cases}$$

then the efficient allocation verifies $R_{i+1}^\star \geq R_i^\star$ and $\theta_{i+1}^\star \leq \theta_i^\star \quad \forall i$.

Proof. Substitute ψ by $1-\gamma$ in proof of property 2, and the proof is straightforward. □

Property 5. *Let $\eta(\theta_i^\star) = 1-\psi$, if $\gamma = 1-\psi$ then $R_i = R_i^\star$ et $\theta_i = \theta_i^\star$ $\forall i$.*

Proof. The proof is straightforward by substituting ψ by $1-\gamma$ in proposition 4 and compare with proposition 2. □

As in Mortensen et Pissarides [1994], the equilibrium is in general not optimal. This is only the case if the non-generic Hosios applies (property 5). Our life cycle economy indeed does not introduce any additional source of externalities, and generations are not overlapping. For the same reason, allowing for endogenous search effort leads to similar optimality results (proof available upon request).

It should be noticed that this result could also have been obtained by maximizing the value of unemployment which can be written in equilibrium as:

$$\mathcal{U}_i = b + \frac{\gamma}{1-\gamma} c\theta_i + \beta \mathcal{U}_{i+1}$$

Let us reason by backward induction and maximize this expression subject to equilibrium definition given by proposition 2, we get $\gamma = 1-\psi$ (if $\eta(\theta_i) = 1-\psi$).

2.3.2 Competitive Search Equilibrium

Lastly, this efficient result can be derived in a competitive search equilibrium context (*à la* Moen [1997]). This consists in allowing for a complete set of markets for each age: both firms and workers enter a particular sub-market that provide a couple (γ, θ_i) so that it maximizes:

$$\begin{aligned} V_i &= \max_{\gamma,\theta_i} \{-c + \beta q(\theta_i)(1-\gamma)(\bar{\epsilon} - R_{i+1})\} \\ \mathcal{U}_i &= \max_{\gamma,\theta_i} \{b + \beta p(\theta_i)\gamma(\bar{\epsilon} - R_{i+1}) + \beta \mathcal{U}_{i+1}\} \end{aligned}$$

A competitive search equilibrium gives couples (γ, θ_i) then satisfying the following condition:[9]

$$\left.\frac{\partial \gamma}{\partial \theta_i}\right|_{V_i} = -(1-\gamma)\frac{q'(\theta_i)}{q(\theta_i)} = -\gamma \frac{p'(\theta_i)}{p(\theta_i)} = \left.\frac{\partial \gamma}{\partial \theta_i}\right|_{U_i} \tag{13}$$

[9] See Mortensen and Pissarides [2000] for more details on competitive search equilibrium derivation.

from which we obtain $\gamma = 1 - \psi$ (if $\eta(\theta_i) = 1 - \psi$).

Overall, both these results suggest that it is efficient to discriminate against older workers by providing them a lower probability of hiring and a higher probability of firing.

2.4 The Incidence of Retirement Age: A Quantitative Illustration

This section aims at addressing the quantitative incidence of workers' horizon on the labor market (retirement age) on hirings, firings and employment rates. Our goal is to show to what extent the retirement age is preponderant in explaining labor market features by age rather than to test model's ability to match data.

To undertake this illustration, we follow Mortensen and Pissarides [2000] by considering properties of the efficient equilibrium model and simulate the model with and without endogenous search effort defined by properties 2 and 3, respectively. This means that in our quarterly calibration we set $\gamma = \psi = 1/2$. Accordingly, corollaries 2 and 3 apply. Furthermore, we set $\beta = 0.99$ and normalize $\bar{\epsilon} = 1$. The opportunity cost of employment, b, is then calibrated so that the model replicates an employment rate of 80% for the age group 25-49, which corresponds to an average over OECD countries (see table 1).[10] This implies $b = 0.308$ for the benchmark model and $b = 0.4688$ for the model with endogenous search effort. Lastly, setting $c = 1$ then implies that $c/q(\theta_i) \in [0, 0.5] \;\; \forall i$ (whatever the model specification), a value to compare with the one assumed in the infinite live agents model of MP, $c/q(\theta) = 0.3$.

2.4.1 Sensitivity of Job Creation and Job Destruction to Retirement Age

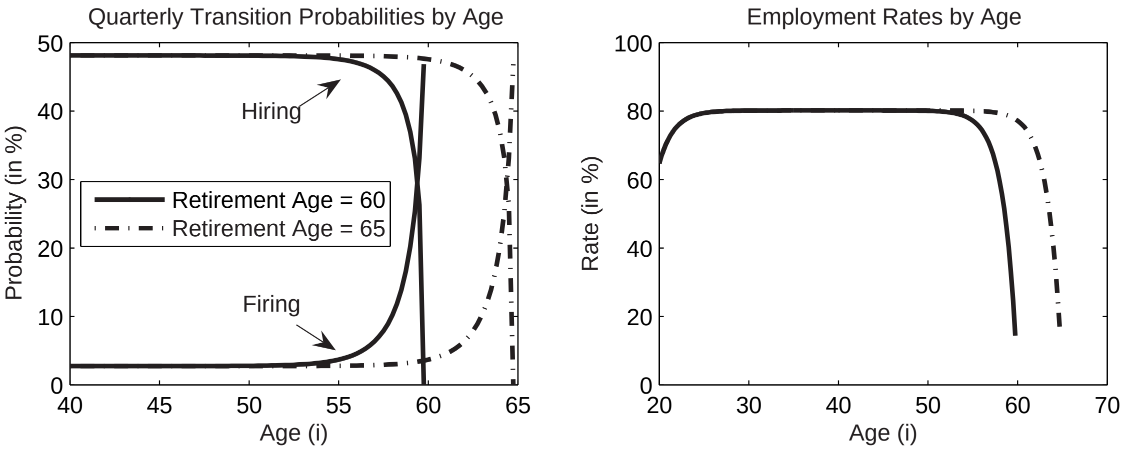
We first look at hirings and firings age properties according to two representative retirement ages, either 60 or 65 (see figure 1). For instance, while in the former case the probability to enter employment for 59 years old workers is falling to zero, average duration of unemployment remains around 6 months for such workers in the latter case. The opposite holds for firings, 59 years old workers expect to be fire in the following quarter with a probability around 1/2 when the retirement age is 60, whereas this quarterly probability

[10] As it will be shown in our quantitative analysis this age group employment rate is not sensitive to the retirement age.

is less than 5% when the retirement age is 65. This clearly shows than hirings and firings of older workers highly depend on this retirement age.

Figure 1 also shows the hump-shaped feature of employment rate according to workers' age. It also emphasizes as a result of hirings and firings profile that employment rate of 59 years old worker is for instance fourth higher when $T = 65$ than when $T = 60$.

Figure 1: Employment rate of 50-64 age group: model's predictions

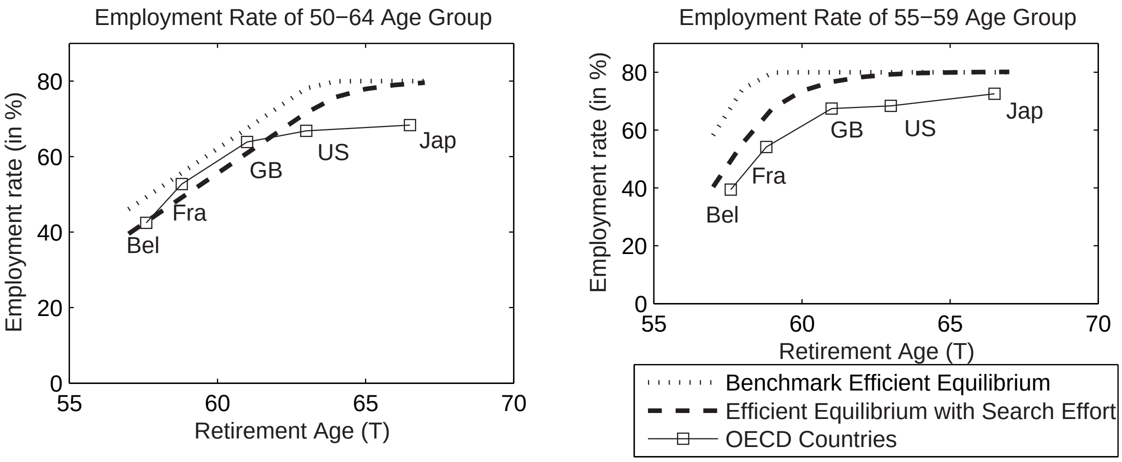


2.4.2 Sensitivity of Employment Rates by Age Groups to Retirement Age

Figure 2 provides additional information by plotting model's employment by age groups predictions according to retirement ages that cover most of OECD countries. We consider models' implication with and without endogenous search effort. Interestingly, our simulations of employment rate of age group 55-59 according to T shows that approximately half of the decrease in the employment rate (with respect to 25-49 reference) is explained by our benchmark model and the other half relies on the conventional search effort "story". This gives some supports to our investigation of life cycle labor market which integrates both labor demand and labor supply factors.

By the way, it is also interesting to note that the model produces realistic employment rate values. But importantly, it is obvious that our goal is not to say that our efficient model is able to match data since various labor market policies such as employment protection and legislation prohibiting age discrimination are implemented in number of these countries.

Figure 2: Employment rate of 50-64 age group: model's predictions



3 Labor Market Policies Revisited

In general (if markets are incomplete), efficiency is not achieved. In addition, welfare state economies allow for unemployment benefit (UB) system more or less generous. The latter induced distortions on job creation and job destruction margins that also leave a room for labor market policies.

We thus allow our model to include firing taxes and hiring subsidies in order to address the question of the optimal design (by age) of these policy tools. We examine the policy incidence of search externalities when they are not completely internalized at the equilibrium. We then turn to the incidence of unemployment benefits[11]. Overall, the design of hiring subsidies and firing taxes is key related to the value of worker's bargaining power and the level of unemployment benefits. In particular, we show that firing taxes and hiring subsidies typically increase with age in a "US type economy" with low UB, whereas they decrease with age in a European one with high UB.

The first best labour market policy requires to differentiate by age employment protection and hiring subsidies. Since it is likely difficult to implement such complex instruments in real world, we lastly look at the impact of implementing a legislation that simply forbid directed search. An illustrative simulation shows that this second best policy is welfare improving for the US but welfare decreasing for the Europe.

[11]It is implicitly assumed that a non-distortionary tax allows to finance the unemployment benefit system.

3.1 First best policies

We now extend our model to account for unemployment benefit system whose financing is allowed by a non-distortionary tax. To offset distortions on job creation and job destruction related to the unemployment compensations and search externalities (if not completely internalized) we consider that the policy tools are employment protection and hiring subsidies.

Let z be the unemployment benefit, F_i the tax that the firm must pay when she fires a worker of age i and H_i the hiring subsidy that the firm gets when she hires a worker of age i, the equilibrium allocation is now featured by (see appendix for details):

Proposition 5. *For given sequences of policy instruments $\{H_i, F_i\}$, a labor market equilibrium exists and it is characterized by $\{R_i, \theta_i\}$ solving:*

$$\begin{aligned} \tfrac{c}{q(\theta_i)} &= \beta(1-\gamma)(\bar{\epsilon} - R_{i+1} + H_{i+1} - F_{i+1}) && (JC_{pol}) \\ R_i &= b + z + \tfrac{\gamma c}{1-\gamma}\theta_i - \beta\left[\textstyle\int_{R_{i+1}}^{\bar{\epsilon}} [1-G(x)]\,dx - F_{i+1}\right] - F_i && (JD_{pol}) \end{aligned}$$

with terminal conditions $R_{T-1} = b + z - F_{T-1}$ and $\theta_{T-1} = 0$.

Proof. See Appendix. □

z is found to play a conventional upward pressure on wages and the productivity threshold R_i, as in MP. Interestingly, whereas F_i tends to push down R_i by increasing the current cost of firing, F_{i+1} instead increases R_i by reducing the value of labor hoarding (the term in brackets).[12]

By comparing these job creation and job destruction rules with the efficient ones it is then straightforward to determine the design of the optimal hiring subsidies and firing taxes.

Proposition 6. *Let $\eta(\theta_i) = -\theta_i q'(\theta_i)/q(\theta_i) = 1-\psi$, the optimal labor market policy is a sequence $\{H_i^\star, F_i^\star\}$ solving:*

$$H_{i+1}^\star = F_{i+1}^\star + \left[\frac{\gamma-(1-\psi)}{(1-\gamma)\psi}\right]\frac{c}{\beta q(\theta_i^\star)} \quad (14)$$

$$F_i^\star = z + \beta F_{i+1}^\star + \left[\frac{\gamma-(1-\psi)}{(1-\gamma)\psi}\right] c\theta_i^\star \quad (15)$$

with boundary conditions $H_{T-1}^\star = F_{T-1}^\star = z$, and where $\theta_i^\star$ is given by the solution of the dynamical system $(JC^\star)$-$(JD^\star)$.

[12] Labor hoarding refers to the expected future gain associated with the job.

Proof. The proof is straightforward by comparing ($JC^{\star}$) and ($JD^{\star}$) with (JC_{pol}) and (JD_{pol}). □

To discuss this outcome, we can disentangle the role played by each distortion, either related to search externalities or unemployment compensations. The two following corollary deal successively with these two sources of distortions and their respective implications on policy.

Corollary 5. *Assume $z = 0$ and proposition 4 is satisfied, the age dynamics of hiring subsidies and firing taxes is characterized by:*

1. *If $\gamma > 1 - \psi$, $H_i^{\star} > H_{i+1}^{\star} \geq 0$ and $F_i^{\star} > F_{i+1}^{\star} \geq 0$.*
2. *If $\gamma < 1 - \psi$, $H_i^{\star} < H_{i+1}^{\star} \leq 0$ and $F_i^{\star} < F_{i+1}^{\star} \leq 0$.*

Proof. Imposing $\theta_{i+1}^{\star} \leq \theta_i^{\star}$ from proposition 4 into proposition 6, the proof is straightforward. □

Assuming $z = 0$ we are focusing on the way to internalize the effects of search externalities. If $\gamma > 1 - \psi$, the worker's bargaining power is higher than its efficient value. This implies that equilibrium wages are higher than would require the optimum. Consequently, there is not enough vacancies at the equilibrium. To correct for this, positive hiring subsidies have to be introduced in order to be consistent with $\theta_i = \theta_i^{\star}$. But at the same time, the large value of γ together with hiring subsidies are responsible for an excessive rate of job destruction: $\frac{\gamma}{1-\gamma} c\theta_i^{\star}$ (from (JD_{pol})) $> \frac{1-\psi}{\psi} c\theta_i^{\star}$ (from ($JD^{\star}$)). This requires to positively tax firings. Until now, the same results would have been obtained in a Mortensen-Pissarides economy with infinite life horizon.

Our additional point is that the size of distortions related to $\gamma \neq 1 - \psi$ is decreasing with worker's age. This is due to $\theta_i \geq \theta_{i+1}$, which indicates that the wage incidence of γ is as less important as worker is old. Ultimately, even if $\gamma > 1 - \psi$, we have $F_{T-1} = H_{T-1} = 0$ (for $z = 0$). Consistently, when $\gamma > 1 - \psi$, we find optimal to reduce the size of employment protection and the amount of hiring subsidies as worker's age is rising up.

In turn, when $\gamma < 1 - \psi$, equilibrium wages are not high enough so that it is optimal to tax hirings and simultaneously encourages firings. For the same reason as before, distortions being lower for older workers, hirings tax and firing subsidies are optimally increasing with worker's age.

Corollary 6. *Let $\eta(\theta_i) = -\theta_i q'(\theta_i)/q(\theta_i) = 1 - \psi$, and assume $\gamma = 1 - \psi$, the age dynamics of hiring subsidies and firing taxes is characterized by $F_i \geq F_{i+1} \geq z$ and $H_i \geq H_{i+1} \geq z$.*

Proof. The proof is straightforward by considering $\gamma = 1 - \psi$ in proposition 6 which implies $F_i^\star = z \sum_{j=0}^{T-1-i} \beta^j$. □

Assuming $\gamma = 1 - \psi$ (search externalities are internalized), we are now focusing on the policy implications of unemployment compensations. The latter simply increases equilibrium wages by the same amount whatever worker's age[13], so that the rate of job destruction is increased and the rate of job creation is decreased. Accordingly, the optimal policy reaction consists in allowing for employment protection and hirings subsidies.

Why does the firing tax decrease with worker's age? Let consider a job with worker of age $T-1$, correct for exceeding wage implies $F_{T-1} = z$. With a worker of age $T-2$, not only z but also F_{T-1} must be internalized: both z, by increasing wage, and F_{T-1} by reducing the value of labor hoarding are found to increase R_i. Accordingly, $F_{T-2} > F_{T-1}$. By backward induction, it thus comes that $F_i^\star = z \sum_{j=0}^{T-1-i} \beta^j$: firing tax internalizes the sum of discounted unemployment compensations flows until exit from labor market.[14] Hiring subsidies are then introduced to avoid the distortion induced by termination costs ($H_i = F_i \ \ \forall i$).

Overall, the age dynamics of firing taxes and hiring subsidies depend both on the value of unemployment benefits and worker's bargaining power. In particular, even though $\gamma < 1 - \psi$, it can be the case that $F_i \geq F_{i+1}$ if the value of z is high enough for the equilibrium wage to be higher than its efficient value. In other words, higher unemployment benefits make more likely a decreasing profile of hiring subsidies and firing taxes by age. On the opposite, if γ and z are low enough, the dynamics is reversed. This can easily be stated in the particular case of $\beta \to 1$.

Corollary 7. *Let $\eta(\theta_i) = -\theta_i q'(\theta_i)/q(\theta_i) = 1 - \psi$, and assume $\beta \to 1$, the age dynamics of hiring subsidies and firing taxes is characterized by:*

- $\left.\begin{array}{l} \textit{if} \quad \gamma \geq 1-\psi \quad \textit{and} \quad z \geq 0 \\ \textit{or} \quad \gamma \leq 1-\psi \quad \textit{and} \quad z \geq \tilde{z} \end{array}\right\}, \quad H_i \geq H_{i+1} \geq z \quad \textit{and} \quad F_i \geq F_{i+1} \geq z$
- $\textit{if} \quad \gamma \leq 1-\psi \quad \textit{and} \quad z \leq \hat{z}, \qquad H_i \leq H_{i+1} \leq z \quad \textit{and} \quad F_i \leq F_{i+1} \leq z$

where $\hat{z} = \left[\frac{1-\psi-\gamma}{\psi(1-\gamma)}\right] c \left[\frac{\psi(1-b)}{c}\right]^{\frac{1}{1-\psi}}$ *and* $\tilde{z} = \hat{z}(1-b)^{\frac{1}{\psi-1}}$.

Proof. See Appendix. □

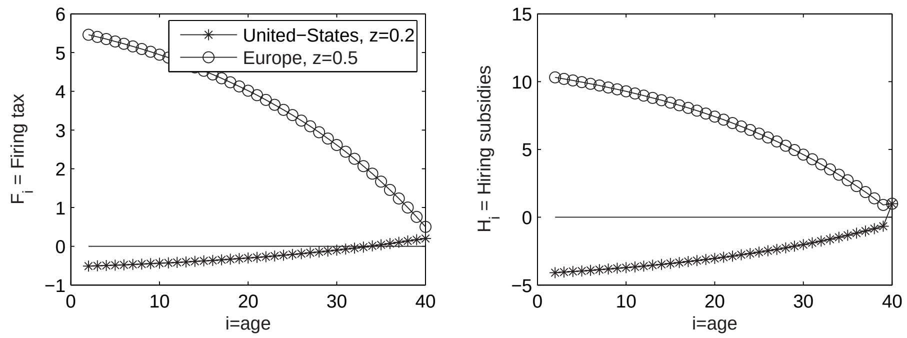
[13] Recall that we assume a non-distortionary tax to finance the UB system.

[14] If it has been assumed that agent have infinite life horizon on the labor market as in Mortensen-Pissarides ($T \to \infty$), it is straightforward to see that $F_i = z/(1-\beta) \ \ \forall i$.

If $\gamma \leq 1 - \psi$ and $z \in [\hat{z}, \tilde{z}]$, the age dynamics of H_i and F_i is typically non-monotonous (first increasing and then decreasing).

This corollary is illustrated in figure 3 which shows that the optimal age dynamics of hiring subsidies and firing taxes in an European economy is just the opposite of the US one. Empirical studies indeed suggest that worker's bargaining power is low (see for instance Abowd and Kramarz [1993] or Cahuc, Postel-Vinay and Robin [2005]) and we set $\gamma = 0.2$ ($< 1 - \psi = 0.5$), so that the profile of H_i and F_i crucially depends on z. Furthermore, it is well known that unemployment benefits are higher in Europe than in the US (see Martin [1999]). Our illustrative quantitative investigation then postulates that US and Europe economies only differ with respect to the value of the replacement ratio ($z = 0.2$ in the US instead of $z = 0.5$ in the Europe). With the calibration reported on the bottom of figure 3, we then have $\hat{z} = 0.24$ and $\tilde{z} = 0.375$. Consistently with corollary 7, it then appears that, in the US, whereas the job of older workers should be protected ($F_{T-1} = z$), incentives should be provided for firms to fire younger workers. On the contrary, in Europe, employment protection should be larger for younger workers than for older workers.

Figure 3: Optimal age dynamics of hiring subsidies and firing taxes in Europe and the US



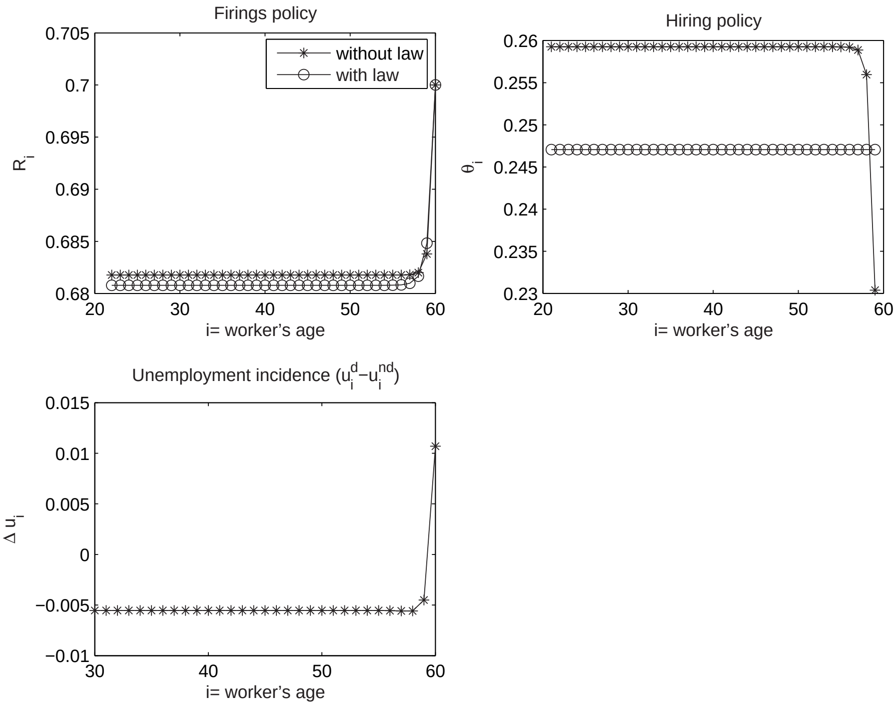
Calibration: $\{\psi = 0.5, \gamma = 0.2, b = 0.2, c = 0.5, \beta = 0.96, \overline{\epsilon} = 1, \mu = 0, u_1 = 1\}$.

3.2 The role of the anti-discrimination law

Implementing the first best policy requires to differentiate hiring subsidies and firing taxes by age. In practice, such design of labor market policy is likely to be difficult to apply. This section wonders to what extent the introduction of a law that forbid directed search (such as in France) could be welfare improving, *i.e.* a *proxy* of the more complex first best policy.

Of course, this law is by itself welfare degrading: the welfare that can be reached is lower than in a *laissez-faire* economy since an additional constraint is introduced. However, in a second best context where search externalities are not internalized and there are unemployment benefits, forbidding direct search might be optimal. The appendix provides a detailed presentation of the model equilibrium when this law applies.

Figure 4: Age dynamics of the labor market: the role of forbidding directed search



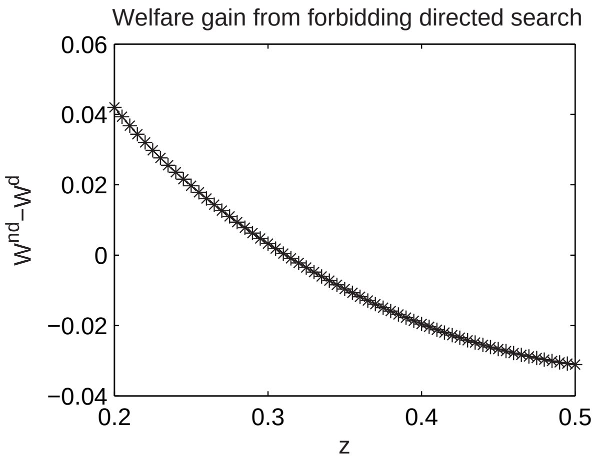
Calibration: $\{\psi = \gamma = 0.5, b = 0.2, z = 0.2, c = 1, \beta = 0.96, \mu = 0, u_1 = 1\}$.

The point is that such a law imposes the same job creation rate whatever the worker's age, which is actually based on an average expected gain. As a consequence, it is favorable to older workers' recruitment as regards to the *laissez-faire* economy (see figure 4). On the opposite, the labor market tightness of younger workers in the *laissez-faire* economy is higher than this average creation rate.

In turn, since wages are positively related with labor market tightness, equilibrium wages and destruction rates of older (resp. younger) workers are increased (decreased). In our simulation exercise the former effect on creations dominates the latter on destructions. In particular, the unemployment rate of older (resp. younger) workers is higher (lower) in a *laissez-faire* economy than in the economy where a law forbids directed search.

Grossly speaking, anti-discrimination law is acting as taxing both hirings and firings of younger workers, with a direct effect on hirings that dominates the indirect one on firings.

Figure 5: Welfare gain from forbidding directed search



Calibration: $\{\psi = 0.5, \gamma = 0.2, b = 0.2, c = 0.5, \beta = 0.96, \mu = 0, u_1 = 1\}$.

Intuitively, the optimality of this law thus depends on the level of unemployment benefits.[15] If the latter are low (as in the US), we showed that it is efficient to tax hirings of young workers and subsidy those of old workers (figure 3.) It is likely that forbidding directed search is optimal in that case. On the contrary, in European countries with high UB, more incentives should be provided for the hirings of young workers than for those of old

[15]To compare equilibrium welfare with and without directed search we use the following definitions, respectively:

$$\begin{aligned} \mathcal{W}^d &= \sum_{i=1}^{T-1} \beta^i \left(y_i^d + b u_i^d - c\theta_i^d u_i^d \right) \\ \mathcal{W}^{nd} &= \sum_{i=1}^{T-1} \beta^i \left(y_i^{nd} + b u_i^{nd} - c\theta^{nd} u_i^d \right) \end{aligned}$$

workers. We may expect that forbidding directed search is welfare degrading in that context. The figure 5 gives an illustrative simulation of that point. It shows with our particular calibration that this second best policy is welfare improving for the US ($z = 0.2$) but welfare degrading for the Europe ($z = 0.5$).

4 Conclusion

This paper originally incorporates life-cycle features into the job creation - job destruction framework. We show the ability of our canonical model to account, at least qualitatively, for the observed drop of older workers' employment rate. This result neither rely on retirement programs nor on productivity/wage decrease. It simply refers to the incidence of age on expected distance from retirement, hence expected duration of jobs.

We then derive normative properties and show in particular that the profile of labor demand policies with age should differ among countries according to differences in unemployment benefit institutions. While in a US type economy hiring subsidies and firing taxes should be more favorable to employment older workers, the reverse holds in european countries with high unemployment compensation.

Overall this paper rehabilitates the life cycle view of labor market, both for understanding supply and demand characteristics, and for implementing welfare-improving policies. Beyond the introduction of more elaborated version of our model with human capital accumulation, a research agenda remains open to precisely investigate our framework's ability to account for life-cycle labor market stylized facts.

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A Extended model with endogenous search effort and human capital accumulation

To Be Completed.

B Proofs of propositions, properties and corollaries

B.1 Proof of proposition 1

By differentiating (3), with $\gamma = 0$, it comes that $J_i'(\epsilon) = 1 \quad \forall i$. Since $J_i(R_i) = 0$, the value of a filled job verifies $J_i(\epsilon) = \epsilon - R_i$. The equation (2) is then rewritten as:

$$\frac{c}{q(\theta_i)} = \beta(\bar{\epsilon} - R_{i+1})$$

This gives immediately $(JC_{PartialEq})$. Since $\int_{R_{i+1}}^{\bar{\epsilon}} J_{i+1}(x)dG(x) = \int_{R_{i+1}}^{\bar{\epsilon}} J_{i+1}'(x)[1-G(x)]dx = \int_{R_{i+1}}^{\bar{\epsilon}}[1-G(x)]dx$ and $V_i = 0 \ \ \forall i$ (free entry condition), the equation (4) verifies $(JD_{PartialEq})$.

B.2 Proof of property 1

The proof is straightforward. Making use of $(JD_{PartialEq})$, we obtain:

$$\begin{aligned} R_{T-1} &= b \\ R_{T-2} &= b - \beta \int_{R_{T-1}}^{1} [1 - G(x)]dx \quad \leq R_{T-1} \\ R_{T-3} &= b - \beta \int_{R_{T-2}}^{1} [1 - G(x)]dx \quad \leq R_{T-2} \\ \ldots \end{aligned}$$

As can be seen in $(JC_{PartialEq})$, θ_i depends negatively on R_{i+1}, and it turns out that $\theta_{i+1} \leq \theta_i \ \ \forall i$.

B.3 Proof of proposition 2

By differentiating (3) with respect to ϵ, it now appears that $J_i'(\epsilon) = 1 - \gamma \ \ \forall i$. Since $J_i(R_i) = 0$, the value of a filled job $J_i(\epsilon)$ is equal to $(1 - \gamma)(\epsilon - R_i)$. The equation (2) can then be re-written in order to determine the sequence of θ_i as an expression of R_i (equation (JC)). Moreover, by combining the wage equation (8) and the equation (4), and integrating by parts as in proof of proposition 1, one gets the equation (JD) describing the age profile of R_i.

B.4 Proof of property 2

If $\frac{dR_i}{dR_{i+1}} \geq 0$ [condition (C1)] and $R_i \leq b$ [condition (C2)] the solution to the time equation (9) necessarily verifies $R_{i+1} \geq R_i$. Given the definition of (JC) and $q'(\theta_i) < 0$, it is possible to show that $\theta_{i+1} \leq \theta_i$. Making use of (9), one can write:

$$\frac{dR_i}{dR_{i+1}} \geq 0 \iff 1 \geq \left(\frac{\gamma}{1-\psi}\right) \left[\frac{\beta(1-\gamma)}{c}\right]^{\frac{\psi}{1-\psi}} (1 - R_{i+1})^{\frac{2\psi-1}{1-\psi}} \tag{16}$$

$$R_i - b \leq 0 \iff 1 \geq 2\gamma \left[\frac{\beta(1-\gamma)}{c}\right]^{\frac{\psi}{1-\psi}} (1 - R_{i+1})^{\frac{2\psi-1}{1-\psi}} \tag{17}$$

If $\psi \geq 1/2$, the condition (C1) for $R_{i+1} = 0$ est une condition suffisante assurant que (C1) est vérifiée $\forall R_{i+1}$ et implique que (C2) est nécessairement vérifiée. A l'inverse, si $\psi \leq 1/2$, la condition (C2) vérifiée pour $\max\{R_i\} = b$ implique que (C1) l'est nécessairement.

B.5 Proof of property 3

Raisonnons par l'absurde. Si $u_1 < \Psi(R_2, \theta_1)$, puisque $\Psi(R_{i+1}, \theta_i) \leq \Psi(R_{i+2}, \theta_{i+1})$ on a alors nécessairement $n_{i+1} \leq n_i \quad \forall i$. Dans ce cas on a donc $u_2 < u_1$, alors que parallèment $\Psi(R_2, \theta_1) < \Psi(R_3, \theta_2)$, dont on déduit $u_3 < u_2$ et $\Psi(R_3, \theta_2) < \Psi(R_4, \theta_3)$, etc...

A l'inverse si cette restriction sur u_1 n'est pas vérifiée, alors $n_2 > n_1$, et $\Psi(R_{i+1}, \theta_i) < \Psi(R_{i+2}, \theta_{i+1}) \quad \forall i$ assure qu'il exsite un age $\tilde{T}$ définit par $u_{\tilde{T}} = \frac{G(R_{\tilde{T}+1})}{G(R_{\tilde{T}+1}) + p(\theta_{\tilde{T}})}$, tel que $n_{\tilde{T}+1} \leq n_{\tilde{T}}$.