Peer Group Composition with Intrinsically Motivated Agents

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Abstract

This paper considers the impact of a peer group's composition on the self esteem and intrinsic motivation of agents. In a model, where agents have imperfect self knowledge we argue that the optimal composition is not only determined by complementaraties in the production function, but also by the agent's intrinsic motivation, which differs for homogeneous and heterogeneous groups. While, low ability agents have a higher motivation in heterogeneous groups, high ability ones are more motivated in homogeneous groups. Hence, the principal can influence the agent's motivation by structuring groups, which induces him to chooses more often heterogeneous groups.

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1 Introduction

Decisions how to form groups are often made depending on the existence of complementarities in the joint production function of the group members. However, this neglects that the composition influences the self esteem and intrinsic motivation of individuals. For example, a low ability agent profits from working together with a talented coworker by learning from her how to produce, or by discussing his solution concepts. This does not only increase his productivity, but makes him feel more competent and thus increases his self esteem. In contrast the talented worker can get demotivated by the presence of the low ability peer.
The aim of this paper is to show that the intrinsic motivation differs in homogeneous and heterogeneous groups and that these motivational considerations can lead to recommendations that are opposite of the group composition one would choose under focusing solely on the existence of complementarities in the production function.

The model is as follows. A principal employs four agents, who work toegther in groups of two agents. Two of them have a high ability and two a low ability (type). Each agent individually completes a task, whose outcome can be either a success or failure. The success probability depends on three things: unobservable effort, talent and whether or not a new solution concept (an "idea") was developed in a group. Ideas – whose probability to be developed depends on an agent's type – can be interchanged between the members of a group: if one agent has no idea, but his peer has, he can use the idea of his peer to produce. Cross influences in a group arises because the agents in a peer group discuss their solution concepts with each other or develop such solutions together.
While in economics it is usually assumed that agents know their ability, we follow the psychology literature and assume that an agent has imperfect self knowledge. However, he receives a noisy signal about his ability. By this we can model an agent's self esteem - the more self confident an agent is to succeed - the higher his self esteem. Hence, the self esteem can be formalized as the belief about one's probability of success, which depends in our model on the type. The principal learns the type of the agents and assigns them either to two heterogeneous ability peer groups (one high and one low ability type in a group) or to homogeneous ones (one group with two low ability agents and one with two high ability ones).[1]
Since the presence of high and low ability types in one group differs in homogeneous and heterogeneous teams – these teams differ with respect to the probability with which ideas are developed and with respect to the spillover of ideas. For example in a homogeneous high type group the probability that both agents have an idea is higher than in a mixed group. However, there is no spillover in such a situation, since each agent can use his own solution

[1] We assume that agents observe with some probability, whether they are in homogeneous or heterogeneous groups, such that the principal can commit to implement an announced structure.

concept and does not need the one of his peer. Overall, we call this effect "production effect". The composition of a peer group has not only implications for production. Since agents have imperfect self knowledge and the probability of generating ideas depends on the type, ideas do not only help the agents to complete their task successfully, but are a signal about ability. Observing the idea of their peer is an additional signal. Hence, by structuring the peer groups the principal can influence the information agents receive about their ability. When agent's have imperfect self knowledge and are thus uncertain about their success probability, this information influences an agents self esteem: believing that a success is more likely translates into a high self esteem and vice versa.
Psychologists argue that the self esteem of an agent is related to his intrinsic motivation. The latter is defined as the motivation of doing something because it is inherently enjoyable[2] and can be enhanced by the feeling of competence or self-esteem by allowing satisfaction of the individual's need for competence. Some authors define intrinsic motivation explicitly as the belief about the probability of success (self esteem) times the value of obtaining a goal. Therefore, the principal can influence – by influencing the self esteem – the intrinsic incentives of an agent. If the agents have a higher intrinsic motivation, the principal has to pay a lower wage, because agents have better incentives to provide effort. Since a lower wage results in higher profits, the principal likes to select the team structure that induces the highest intrinsic motivation. This effect is called "motivational effect".

Taking the self esteem of the agent's as given and considering only the production side, we show that whether homogeneous or heterogeneous teams are better depends on, in which team ideas are more often produced and more efficiently used. Assume first that agents with different types use ideas equally efficiently. Then the principal prefers heterogeneous teams, since the spillover of ideas is larger in these. However, types differ in their productivity to use ideas: high types can profit more them than low types. This favors homogeneous teams. Overall, in homogeneous team the marginal productivity of an idea is higher, while the spillover of ideas is larger in heterogeneous teams. Taking these two together we show that there is a tendency to separate agents if they are very unequal and to mix them if they are more equal. This result is related to the assortative matching result of Becker (1973). In a market for marriage Becker (1973) shows that if the abilities of men and women are complements in the production function, we should expect positive assortative matching (or a homogeneous peer group), while if they are substitutes a heterogeneous marriage is optimal. In our paper the optimal peer group strcuture does not only depend on the complementarities between types, but also between types and ideas.
Concerning the intrinsic motivation of the agents we argue that the self esteem depends on

[2]An overview of the definitions of intrinsic and extrinsic motivation (doing something because it leads to a separable outcome) or self esteem and how they work together can be found in Ryan and Deci (2000) or in Leonhard, Beauvais and Scholl (1995). The literature on intrinsic motivation was initiated by Deci (1971).

the peer group. Agents observe their own and their peers success in generating ideas and since the probability of having ideas differs in the different teams, the posterior belief differs. We show that the self esteem and hence the intrinsic motivation of a high type is on average lower in a heterogeneous group compared to a homogeneous one, while for a low type it will be higher. Assuming that the principal cannot condition the agents' wage on ideas, we then argue that heterogeneous teams are better from a motivational perspective. This is due to the very low motivation of agents in a homogeneous teams in case no ideas are developed. Hence, taking the motivational and production effect together there is a stronger tendency to implement heterogeneous teams than the production effect suggests.

We adopt the formalization of self esteem and intrinsic motivation from Bénabou and Tirole (2003).[3] In their model[4] an individual has imperfect knowledge about his type or the task's difficulty. He undertakes the task only if he has a high enough self esteem. The principal knows the type and since effort and ability are complements the principal wants to enhance the agent's self esteem. Hence, the wage is a signal about an agent's type and therefore it does not only influence the motivation of the agent directly via the payoff, but also indirectly via the agent's inference process. Bénabou and Tirole (2003) show that rewards are bad news for the agent: the principal offers a equal or lower wage to a more able agent and hence a higher wage reduces the agent's intrinsic motivation. In equilibrium the principal either pools the agents on the lowest possible wage or mixes for the low type between the lowest and a higher one and pays the high type the lowest one.
Besides Becker's (1973) matching model, that asks how to groups agents together according to ability, there is a large literature that considers the question of the optimal team or group structure. However, the dimension according to which agents can be grouped together differ. In Ferreira (2002) this dimension is group loyalty, which is determined by the degree of altruism of an agent, Jeon (1996) and Meyer (1994) consider different generations of agents, Hvide, Fershtman and Weiss (2005) deal with different concerns for status and finally Kräkel (2005) asks for the optimal structure if there are emotions like pride in envy. Glazer and Segendorff (2001) do not ask how a principal wants to group different ability individuals

[3]For a non-technical economic paper about intrinsic and extrinsic motivation see also Kreps (1997). Empirical support for the hidden costs of rewards and intrinsic motivation from economic experiments can be found in Gneezy and Rustichini (2000), who show that higher monetary transfers can lead to a reduction in effort and that there is a non-monotonic relationship between effort and monetary transfers and in Fehr and Gächter (2001), who also observe that material transfers may crowd out cooperation (effort) and that giving no incentives may be better. For psychological experiments Deci, Ryan and Koestner (1999) is a good source. They present a meta-analysis of 128 studies. One of their result is that most kind of rewards (e.g. performance based rewards) undermine intrinsic motivation. For a critical review of these experiments from an economic point of view see Falk and Fehr (2002).

[4]See also Bénabou and Tirole (2002) for a non-technical summary of the model and of concepts like (among others) intrinsic and extrinsic motivation, imperfect self knowledge, self-confidence, self esteem, the looking glass self from an economic point of view.

together, but whether the agent himself likes to work together with a high or a low ability (reputation) individual. Auriol and Renault (2003) consider the question, whether an ex ante homogeneous work force should be made more heterogeneous by giving different status levels to agents.

Most closely related to our model is Battaglini, Bénabou and Tirole (2005). In their model individuals have imperfect self-knowledge. But since they are more interested in peer groups like the anonymous alcoholics, they introduce a self control problem. Observing peers, who are struggling with the same problem as the agent (e.g. alcoholism) is an additional signal about the severity of that problem. So similar to our model peer effects are of an informational nature and have effects on the agent's motivation to perceive a goal. The main difference between our model and theirs is that we introduce a principal, who can influence – by structuring the groups – the information agents receive.[5]

There is a large empirical literature dealing with the question how to compose peer groups. This literature can however not serve as empirical evidence for our results, as we discuss below, although overall there is a tendency for heterogeneous teams, which is conform with our result. Most of the studies consider the effects of a heterogeneous or homogeneous groups on achievement in schools[6], and are thus of limited use to test our results, since we consider extrinsic motivators like wage payments instead of grades. Hamilton, Nickerson and Owan (2003) show that workers' heterogeneity increases output in a firm, which is conform with our result. However, one should note that for the garment plant they consider, developing new solution concepts or imperfect self knowledge about ability should not play a big role compared to complementarities in the production function that lead in the opinion of Hamilton, Nickerson and Owan (2003) to their result. Song (2004) directly examines the effects of peer group composition on intrinsic motivation and finds weak positive effects of heterogeneous groups. He structures peer groups according to self esteem and not to the type, which limits the insights of a comparison of his empirical results, with our model.

The paper is structured as follows. In the first section we will introduce the model. Thereafter we proceed with the analysis and the results. The last section discusses some assumptions and concludes.

[5]Without introducing a principal they also ask about the characteristics of an optimal peer for dealing with self control problems. The optimal one is a weaker one, because this makes success more encouraging and failure less discouraging. In contrast in our model the optimal peer is not necessarily a lower ability agent, neither from a motivational perspective nor from the perspective of a better knowledge transfer.

[6]E.g. Fertig (2003), who finds empirical evidence that heterogeneous peer groups in schools have a reducing effect on individual achievement. He argues that the superiority of homogeneous teams might be due to the higher motivation in such groups. In contrast Robertson and Symons (2003) or Feinstein and Symons (1999) showed that streamed classes have a negative impact on student's achievement. There is also weak evidence for diminishing returns to scale between attainment and the peer group, which is an argument for heterogeneous groups. Other papers like Hanushek, Kain, Markman and Rivkin (2001) found no evidence that changes in the heterogeneity of students affect the average achievement.

2 The Model

There is one principal, who employs four agents. The type of the agent or his ability θ, can be either high (θ_H) or low (θ_L) with $\theta_H > \theta_L$. There are two agents of type θ_L and two of type θ_H. Agents are protected by limited liability and for simplicity we assume that agents always participate. By this we can focus on the incentive effects of a team structure.
Each agent separately produces an observable and verifiable output, $y \in \{0, 1\}$. The probability that output is high depends three variables. First of all on effort e, which can be either high – $e = 1$ – which costs the agent c, or low – $e = 0$ – which costs 0. Second an agent's ability $\theta \in \{\theta_L, \theta_H\}$ enters. Third the agent's success probability is higher if he can use a new solution concept or "an idea" (outcome Y) to produce, compared to if he has no such idea (outcome N). The probability that an agent develops an idea depends in the following way on an agent's type: $1 > \Pr(\mathcal{S} = Y|\theta_H) \equiv p > \Pr(\mathcal{S} = Y|\theta_L) = 0$. Thus, low types never produce innovative solution concepts.
Summarizing, the probability that an agent produces a high output is:

$$\Pr(y = 1|\theta, \mathcal{S}, e) = ef(\theta, \mathcal{S}) = e\theta q(\mathcal{S}),$$

with $f : \Theta \times S \times e \to (0, 1)$, $\mathcal{S} \in \{Y, N\}$, $e \in \{0, 1\}$. We express with the assumption $\Pr(y|\theta, \mathcal{S}, 0) = 0$ that effort and talent are complements. For example ability is not enough to succeed - if the agent provides no effort he will fail, no matter how talented he is. We let $q(Y) \equiv \bar{q} > q(N) \equiv \underline{q}$. Hence, our probability of success function satisfies:

(A1) $f(\theta, Y) > f(\theta, N) \; \forall \; \theta \in \{\theta_L, \theta_H\}$.

(A2) $f(\theta_H, \mathcal{S}) > f(\theta_L, \mathcal{S}) \; \forall \; \mathcal{S} \in \{Y, N\}$.

(A3) $f(\theta_H, Y) - f(\theta_H, N) > f(\theta_L, Y) - f(\theta_L, N)$.

$(A1)$ states that a new solution concept leads to a higher success probability. Such a concept improves the way an agent produces or helps the agent to understand better the problem he has to solve. From $(A2)$ we see that high ability agents are more productive than low ability agents. The third assumption says that ability and ideas satisfy strictly increasing differences, which implies that these two are complements. Hence, high ability agents make more out of new solution concepts than their low ability colleagues. For example, they better understand how to apply an idea to solve a problem.
The completed production has a nonmonetary value yV to an agent and yW to the principal, with $y \in \{0, 1\}$ and $W > V > 0$. The value V captures the intrinsic gain of an agent, that he gets in case he succeeds in his task. For example he can feel proud or happy if he did a good job and these feelings increase his utility.[7] Concerning the gain of the principal and agent we make the following assumption:

[7] One can endogenize the value V by career concerns or interpret V as human capital.

Assumption 1 $(1-p)f(\theta_L, N)(W+V) - c > 0$, *and* $V - c < 0$.

Hence, the expected total value of completing the task successfully is from an ex ante perspective high enough for a low type, although no idea is generated. This implies that the total value is high enough for a high type or if an idea was generated. Thus, the total profit is maximized if an agent provides high effort, although it is ex ante not clear, whether he produces an idea. However the intrinsic value an agent gets out of a project is never high enough to induce him to work.

The timing and information structure of the model are as follows. At the first stage the principal announces, how he groups agents together and offers a wage scheme to the agents. We understand a peer group as a loose union of two agents - e.g. two colleagues sitting together in a large office and discussing ideas with each other, two agents that are working on a similar task or project, a school class or a study group. Peer groups $i \in \{hom, het\}$ can be either homogeneous (a group consisting of two $\theta_L\theta_L$ agents and one with $\theta_H\theta_H$), which we denote by hom or heterogeneous (two $\theta_L\theta_H$ and $\theta_L\theta_H$ groups), which we denote by het.[8] The wage scheme specifies that the agent receives a positive bonus b_i in case he succeeds and nothing else. In section 4 we show that – given our assumptions – that the wage scheme does indeed not condition on $\mathcal{S}$ or an agent's type and discuss what happens if we relax the assumption that an agent's wage does only condition on his output and not on the one of the colleague. At this stage neither the principal, nor the agents know the types of the agents.
At the second stage the principal learns the agents' types, while the agents do not. Thus, they have imperfect self knowledge. For example the principal knows better the difficulty of a task and the agent's talent to perform this task than the agent. The prior probability of an agent to be of type θ_H is $\lambda = \frac{1}{2}$. After learning the types, the principal assigns agents to peer groups (teams) of two agents. We assume that the principal can commit to implement the team structure he announced at stage 1. We discuss this assumption later on.
At the third stage agents produce ideas,[9] which are not verifiable to the principal. For example, compared to output ideas are a more vague concept and it is hard to specify ex ante what a good idea is. We assume that an agent can use if he was unsuccessful in generating an idea, the idea of his teamcolleague, i.e. we assume that ideas are nonrival and observable in teams: the colleagues in an office discuss their solution methods with each other or students

[8] We will restrict our attention to wage schemes that induce pure strategies at the team formation stage, i.e. we will not consider wage schemes for which the principal is (non generically) induced to mix between team structures. Although such wage schemes can for some parameters lead to higher expected profits, we are more interested how the realized team structure looks like. We discuss mixed strategy equilibria in the appendix.

[9] We assume that no effort is needed to produce them. If effort would be needed an agent might handicap himself and prefer not to provide effort to avoid the information about his type. Although this is an interesting aspect, it is not our topic and would make our model much more complicated. See e.g. Stone (2004) for a model (on a different topic) with self handicapping.

observe in a discussion in class the ideas of the others, which they can then apply to solve their own task. Whether or not an idea is developed in one group can not be observed in the other group.

Since the agents have imperfect self knowledge and the probability of developing an idea depends on their ability, they update their prior about being the high type after observing their own success in generating ideas and the one of their colleague. Hence, they believe that their success probability is $E[f(\theta, \mathcal{S})|\mathcal{I}]$, where $\mathcal{I} = (\mathcal{S}, i)$ and $E[f(\theta, \mathcal{S})|\mathcal{I}] = \Pr(\theta_H|\mathcal{I})f(\theta_H, \mathcal{S}) + \Pr(\theta_L|\mathcal{I})f(\theta_L, \mathcal{S})$. We relate this posterior belief about one's success probability to an agent's self esteem. Agents, who believe that a success is more likely have a higher self esteem than those agents, who think that they fail. Especially, the self esteem is increasing in the probability with which the agent beliefs to be the high type: having a positive self image and thinking that one is talented is associated with a high self esteem. But as we discussed in the introduction a high self esteem results in a higher intrinsic motivation, which is decomposed of the value of obtaining a goal (here V) and the self esteem. Thus, the agent's expected intrinsic value – $E[f(\theta, \mathcal{S})|\mathcal{I}]V$ – is a way to capture the intrinsic motivation of an agent: as we discussed in the introduction, the intrinsic motivation is composed of the value of a goal and the self esteem and will be higher, the higher the self esteem is.

At the last stage agents provide unobservable effort e, which helps them besides their ideas and talent to complete the task successfully. Finally output and payoffs realize. The principal receives for each successful agent W and pays the specified bonus, while an agent receives the bonus, his intrinsic gain if he is successful minus the effort costs if he provides high effort. The aim of the principal is to maximize his profits.

3 Analysis

3.1 Basic Effects

In this section we first analyze how homogeneous and heterogeneous groups differ according to the creation and spillover of ideas (i.e. the production effect) and second to the intrinsic motivation of the agents.

3.1.1 Production Effect - Effort Observable

To separate any motivational considerations from production effects, we abstract from incentive and self esteem problems in this section and assume that effort is observable and that the principal as well as the agents know the agent's type. Hence, the principal pays an agent $c - \Pr(success)V$ if he provides effort and zero else (assuming whenever indifferent the agent provides effort).

The principal's expected profit for a homogeneous team is then given by $2[(2p - p^2)f(\theta_H, Y) +$

$(1-p)^2 f(\theta_H, N) + f(\theta_L, N)](W+V) - c$. Low types never develop ideas and hence succeed with probability $f(\theta_L, N)$, while the probability that both high types do not have an idea is $(1-p)^2$. In this case high types succeed with probability $f(\theta_H, N)$. With probability p^2 both high types find a innovative solution concept and with $2p(1-p)$ only agent finds one, which however the peer can use. In these cases the probability of success is $f(\theta_H, Y)$. For heterogeneous groups the expected profit is $2[p\{f(\theta_H, Y) + f(\theta_L, Y)\} + (1-p)\{f(\theta_H, N) + f(\theta_L, N)\}](W+V) - c$. The high type develops an idea with probability p. This idea can also be used by the low type. With $(1-p)$ he finds no solution concept and the high as well as the low type have to produce without such a concept.
Subtracting the expected profit for heterogeneous teams from the one for heterogeneous teams and rearranging Lemma 1 follows:

Lemma 1 *Suppose that effort is observable and that all players know the agents' types. Then the principal prefers homogeneous teams to heterogeneous iff:*

$$\{p(\theta_H, \theta_H) - p(\theta_H, \theta_L)\}\{f(\theta_H, Y) - f(\theta_H, N)\} > \{p(\theta_L, \theta_H) - p(\theta_L, \theta_L)\}\{f(\theta_L, Y) - f(\theta_L, N)\}, \tag{S}$$

where $p(\theta_H, \theta_H) = 2p - p^2$, $p(\theta_L, \theta_H) = p(\theta_H, \theta_L) = p$ *and* $0 = p(\theta_L, \theta_L)$.

To illustrate the Lemma, suppose first that $f(\theta_H, Y) - f(\theta_H, N) = f(\theta_L, Y) - f(\theta_L, N)$. Then, whether homogeneous or heterogeneous teams are better depends on whether types are complements or substitutes, as it was shown by Becker (1973).[10] For the specified probability to generate an idea it turns out that equation (S) reduces to $(1-p) < 1$, implying that types are substitutes and hence heterogeneous teams are better. This is due to the higher spillover of ideas in heterogeneous teams: whenever a high type has an idea, he passes it on to his low type peer. While in homogeneous teams no such spillover occurs in a situation, where both high types have an idea or where both agents are of the low type. However, $(A3)$ states that $f(\theta_H, Y) - f(\theta_H, N) > f(\theta_L, Y) - f(\theta_L, N)$, i.e. types and ideas are complements and this favors homogeneous teams, since in these high types can use more often an idea than in heterogeneous teams (where in comparison low types can more often use them) and they use them more efficiently than low types.
Hence, whether homogeneous or heterogeneous groups are optimal depends on the relative strength between the complementarity between ideas and types and the one between high and low types. To see how this depends on the parameters rearrange condition (S) and plug in the values for p. Then homogeneous teams are better iff:

$$(1-p)[f(\theta_H, Y) - f(\theta_H, N)] > [f(\theta_L, Y) - f(\theta_L, N)].$$

Homogeneous teams can be better if p is not too large or the increase in the success probability for high types, who can use an idea compared to not is much larger than this increase

[10] See Sundaram (1996) that iff $\frac{h(\theta,\theta')}{\partial\theta\partial\theta'} \geq 0$ then $h(\theta, \theta')$ satisfies increasing differences, where in our context $h(\theta, \theta') = p(\theta, \theta')g(\theta)$.

for low types. The latter statement implies that types have to be very different. As the difference between the types increases, p can increase. Thus, the larger the spread in abilities or the smaller the spread in the success probabilities in generating an idea for high and low types, the stronger is the tendency to separate types. I.e. homogeneous groups are more likely to be optimal if agents are very different in their ability to use ideas or the spillover of ideas in heterogeneous teams is small, while heterogeneous groups are optimal if they are more similar or for a large spillover.

3.1.2 Intrinsic Motivation

If agents do not know their types, the composition of peer groups has also implications for the self esteem and the intrinsic motivation of the agents, which is given by the posterior expectation about their success times the value of the goal – $E[f(\theta, \mathcal{S})|\mathcal{I}]V$, with $\mathcal{I} = (\mathcal{S}, i)$. In this section we want to examine solely this intrinsic motivation - abstracting from the effort choice.

Since the signals that agents receive differ in heterogeneous and homogeneous groups, the self esteem and intrinsic motivation differ between groups. From Table 1 we see that the high type agent learns his type whenever he or his peer develop an idea (states YY, YN, NY). Thus, the agent has a very high self esteem in these states. In comparison the high type in a heterogeneous team has such a high self esteem only in state YN, i.e. less often than in a homogeneous team. However, the high type in a homogeneous group has a lower self esteem when neither he nor his colleague had an idea than the high type has in a heterogeneous team, since $\frac{1}{2} \geq \Lambda$, $\Lambda \in [0, \frac{1}{2}]$. This is driven by the fact that the outcome NN is much more likely for two low ability agents than for two high types or a high and low type.

The low type's posterior to be of type θ_H in a homogeneous team is always Λ, since both agents never develop an idea. Hence, agents in this groups have a very low self esteem, because they can never use an idea and moreover, get a strong signal about being untalented. As for the high type, this self esteem is lower than the one in a heterogeneous group in state NN. However, the self esteem can be higher or lower than the one in state NY. On the one hand the low type agent in a heterogeneous group learns in state NY that he is the low type. This lowers the self esteem. But on the other hand he can use the good idea of his peer, which makes him feel more competent.

What about the average motivation of a high and low type in heterogeneous versus homogeneous teams? In the appendix we show that:

Proposition 1 *The intrinsic motivation of a high type is on average higher in a homogeneous group. The one of a low type is higher in a heterogeneous group.*

High types have a higher intrinsic motivation in homogeneous groups, since they receive more often positive signals about their ability – that increase their self esteem – than in a heterogeneous group. This offsets the negative effect of having a very low motivation in case

Event / $\Pr(\theta_H \mid JK, i)$	Homogeneous Groups	Heterogeneous Groups
YN	1	1
NY	1	0
YY	1	not defined
NN	$\frac{(1-\bar{p})^2}{(1-\bar{p})^2+1} \equiv \Lambda$	$\frac{1}{2}$

Table 1: Posteriors after Stage 3. $J,\ K \in \{Y, N\}$

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none agent developed a new solution concept. In comparison low types receive more positive signals than in heterogeneous teams. Hence, overall there is a trade-off between motivating high and low types.

The result has for example implications for the average effort of students in schools. To illustrate this assume that agent's effort is a continuous variable $e \in I\!R^+$ and an agent's cost function is quadratic, i.e. $c(e) = \frac{e^2}{2}$. Furthermore, the only source of motivation is an agent's intrinsic motivation – there are no extrinsic motivators like wage payments. Hence, the agent maximizes: $\max_{e \in I\!R^+} eE[f(\theta, \mathcal{S})|\mathcal{S}, i]V - \frac{e^2}{2}$, which results in in state $\mathcal{S}$ in the effort choice $e^\star = E[f(\theta, \mathcal{S})|\mathcal{S}, i]V$. Thus, the higher a student's intrinsic motivation, the more effort he will provide, which makes a success more likely. Since the effort choice of the agent exactly mirrors the student's intrinsic motivation, we can apply Proposition 1 to see that low types provide more effort in heterogeneous groups than in homogeneous ones, while for high ability students it is exactly the other way round.

3.2 Types Unknown

We solve backward - first considering the agent's incentives to provide effort and then the principal's choice of the bonus b_i and the team structure (where we consider only the pure equilibria cases).

3.2.1 Stage 3 and 4 - Posterior and Incentives of an Agent

Note that the agent does not know his type or the one of his peer, but the peer structure. Hence he forms beliefs at stage three about his type, given the information he has. Since (as we show later) the bonus b_i the principal pays does not condition on an agents type, this information consists of the realization of the idea generating process of the agent and his peer and – since the peer structure is observable – of the peer structure, the principal implements.

Given this information, the agents provides high effort at stage four in state $\mathcal{S}$ iff the following incentive constraint holds:

$$E[f(\theta, \mathcal{S})|\mathcal{I}](V + b_i) - c \geq 0,$$

where $\mathcal{I} = (\mathcal{S}, i)$, $i \in \{hom, het\}$ and $\mathrm{E}[f(\theta, \mathcal{S})|\mathcal{I}] = \Pr(\theta_H|\mathcal{I})f(\theta_H, \mathcal{S}) + \Pr(\theta_L|\mathcal{I})f(\theta_L, \mathcal{S})$, is the expected self esteem that individuals developed at stage 3.

3.2.2 Stage 1 and 2 - Wage Scheme and Peer Group Structure

At stage 2 the principal learns the types of the agents and assigns them to the teams. As we have assumed he can commit to the announced peer structure, he implements the structure he announced at the first stage.

At stage 1 the principal selects the wage scheme for homogeneous and heterogeneous teams that maximizes his profits for this team. Furthermore, he chooses to announce that he will implement the team structure that yield higher profits given the specified bonus.

We first consider the choice of the bonus for homogeneous and heterogeneous teams. Note that the posterior $E[f(\theta, \mathcal{S})|\mathcal{I}]$ differs between the different states of a world as we have seen in section 3.2. If the principal wants to induce the agents to work in all states of the world he has to pay a bonus that induces the agent to work also in the state, where he has the lowest self esteem. For homogeneous teams this is state NN. For heterogeneous teams it can be either state NY or NN depending on, whether $f(\theta_L, Y)$ is smaller or larger than $\frac{1}{2}(f(\theta_H, N) + f(\theta_L, N))$. We first consider the case, where $f(\theta_L, Y) > \frac{1}{2}(f(\theta_H, N) + f(\theta_L, N)$. In the appendix we show that:

Lemma 2 *The principal prefers that the agents work in all stated of the world.*

Hence, he pays the agents:

$$b_{hom} = \frac{c}{\Lambda f(\theta_H, N) + (1-\Lambda) f(\theta_L, N)} - V, \quad \text{and} \quad b_{het} = \frac{c}{1/2 f(\theta_H, N) + 1/2 f(\theta_L, N)} - V.$$

Note that $b_{hom} > b_{het}$, because the motivation of a low type agent is always lower in a homogeneous team.

Given these wages we now derive, which structure the principal likes to implement. Let$\Pr_{hom} = (2p - p^2)f(\theta_H, Y) + (1-p)^2 f(\theta_H, N) + f(\theta_L, N)$ and $\Pr_{het} = p[f(\theta_H, Y) + f(\theta_L, Y)] + (1-p)[f(\theta_H, N) + f(\theta_L, N)]$ the overall success probabilities for heterogeneous and homogeneous teams respectively. We then see that the principal prefers homogeneous groups to heterogeneous ones iff:

$$\Pr_{hom}(W - b_{hom}) > \Pr_{het}(W - b_{het})$$

$$\leftrightarrow \qquad \underbrace{(\Pr_{hom} - \Pr_{het})}_{\text{Gain: higher success probability}} \qquad W > \qquad \underbrace{\Pr_{hom} b_{hom} - \Pr_{het} b_{het}}_{\text{Loss: higher expected bonus}} \quad . \tag{2}$$

The possible gain for homogeneous teams might be a higher success probability, whenever $\Pr_{hom} > \Pr_{het}$. This is the production effect. However there is a loss, namely a loss from the reduced motivation, that is captured in a higher bonus, which is the motivation effect. Whether this motivational effect favors heterogeneous or homogeneous teams depends on, whether the right hand side of Equation 2 is larger or smaller than zero. In the appendix we show the following Proposition:

Proposition 2 *Let $f(\theta, \mathcal{S}) = \theta q(\mathcal{S})$ and assume that $q(Y)\theta_L > \frac{1}{2}q(N)(\theta_L + \theta_H)$. Then the motivational effect always favors heterogeneous teams and hence there is a stronger tendency to implement heterogeneous teams than under the production effect.*

This is caused by the higher motivation of agents in heterogeneous teams. Exploiting this, the principal can reduce the ex ante rent he has to pay to the agents.

4 Discussion

4.1 Commitment

In this section we discuss the assumption that the principal can commit to implement the announced team structure. If the principal wants to implement heterogeneous teams such a commitment is always possible. This is because state YY only occurs if he implements homogeneous teams. Hence, the principal can specify in the contract a payment to the agents for the case that he announces heterogeneous teams and implements then homogeneous teams. Agents anticipate that they detect a possible deviation from his announcement with probability $(1-p)^2$. Hence, the expected (with probability $(1-p)^2$) payment has to be higher than the gain from deviating at stage two.
If he announces homogeneous teams and implements heterogeneous teams at stage 2, the agents could not detect in any state that the principal deviated. Hence, to justify the assumption that the principal can commit we have to assume an that there is an exogenously given small probability with which the agents learn the implemented team structure to make a commitment possible.[11]

4.2 Wage Scheme

So far we assumed that the principal pays a bonus to the agent in case he succeeds. In the appendix we show that the principal can not commit to offer the agents wage schemes of the form "if I learn at stage 2 that you are type θ_H and you succeed I pay you $b(\theta_H)$ and if I learn that you are θ_L I pay you $b(\theta_L)$, with $b(\theta_H) \neq b(\theta_L)$". Thus, wages can not serve to separate agents.
The other assumption we made concerning the wage scheme was that an agent's wage does only condition on the agent's output not on the one of the peer. For heterogeneous teams this is without loss of generality, because the agent's type and hence success probabilities are uncorrelated. In homogeneous teams types are however (positively) correlated, which implies that the principal would like to implement a more competitive wage scheme, rewarding the

[11] One example for this is that there are different tasks for heterogeneous and homogeneous groups and that agents can observe and distinguish the tasks. By this they can also observe the team structure. Another assumption is that the principal has a reputation for building a specified team structure.

agent only if he succeeds and the colleague fails. Since, we wanted to consider the impact of only the intrinsic motivation of the agents on the choice of the peer group, we subtracted from this "competition" effect by assumption.
Lastly we followed Bénabou and Tirole (2003) and specified that the wage scheme is of the form of a bonus, i.e. the agent gets not rewarded if he fails.

5 Conclusion

In this model we have seen that the decision whether to implement heterogeneous or homogeneous peer groups does not only depend on spillover effects, but also on motivational ones. High ability types have a higher intrinsic motivation in homogeneous groups, while low ability types have a higher one in heterogeneous ones. In case the principal can commit to the peer structure we have seen that heterogeneous teams do relatively better than homogeneous teams compared to a situation where only the spillover of ideas matters, due to the high motivation of low types in such teams.

Appendix

Proofs

Proof (Lemma 1).

The result can be easily obtained by subtracting the success probability for heterogeneous teams $p(f(\theta_H, Y) + f(\theta_L, Y)) + (1-p)(f(\theta_H, N) + f(\theta_L, N))$ from the one for heterogeneous ones $(2p - p^2)f(\theta_H, Y) + (1-p)^2 f(\theta_H, N) + f(\theta_L, Y)$. ■

Proof (Proposition 1).

In a homogeneous team the average self esteem of a high type is:

$$(2p - p^2)f(\theta_H, Y) + (1 - p^2)[\Lambda f(\theta_H, N) + (1 - \Lambda)f(\theta_L, N)].$$

and in a heterogeneous team:

$$pf(\theta_H, N) + (1-p)[1/2f(\theta_H, N) + 1/2f(\theta_L, N)].$$

Subtracting the former from the latter:

$$p(1-p)f(\theta_H, Y) + \underbrace{[(1-p)^2\Lambda - 1/2(1-p)]f(\theta_H, N)}_{(1)} + \underbrace{[(1-p)^2(1-\Lambda) - 1/2(1-p)]}_{(2)} f(\theta_L, N)$$

$$(1) = (1-p)\frac{(1-p)^3 - 1/2((1-p)^2+1)}{(1-p)^2+1} = (1-p)\frac{2.5p^2 - 2p - p^3}{(1-p)^2+1}.$$

$$(2) = (1-p)\frac{(1-p) - 1/2((1-p)^2+1)}{(1-p)^2+1} = (1-p)\frac{-1/2p^2}{(1-p)^2+1}.$$

Assume now that $f(\theta_H, Y) = f(\theta_H, N) = f(\theta_L, N)$, i.e. make the first and the second term, which are negative as large as possible compared to the first, which is positive. Then adding all the terms up: $p(1-p)\frac{2-2p+p^2+2.5p-2-p^2-1/2p}{(1-p)^2+1} = 0$.

In a homogeneous team the self esteem of a low type is:

$$f(\theta_L, N).$$

and in a heterogeneous team:

$$pf(\theta_L, Y) + (1-p)[1/2f(\theta_H, N) + 1/2f(\theta_L, N)].$$

Subtracting the former from the latter, making the second term small by taking $f(\theta_L, N) = f(\theta_L, Y)$:

$$1/2(1-p)[f(\theta_L, N) - f(\theta_H, N)] < 0.$$

■

Proof (Lemma 2).

To be added. ■

Proof (Proposition 2).

To be added. ■

Wage Scheme

Proof (Wages not Attached to Types).

Suppose the principal offered a wage scheme of the form "if you are type θ I pay you $b(\theta)$ and suppose $b(\theta_H) \neq b(\theta_L)$. Suppose first that both types provide high effort under their specified bonus. But then the principal would have an incentive to claim that an agent is of the type (in the team), who gets the lower wage.

To be added: separation also not possible if one type of agent does not work in all states of the world under the specified wage. ■

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