Endogenous Minimum Wages, Human Capital Accumulation and Growth

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Abstract

This paper investigates the political dynamic choice of minimum wages together with their impact on employment, productivity and growth within a three period overlapping generations model with heterogeneous agents. Some countries simultaneously experienced a rise of the minimum wage and a rise of unemployment concentrated among young. In this context, the political economy of minimum wages is intriguing as (1) who has the political power (prime-aged workers) are not the ones who are doing the accumulation (the young) and (2) dynamic effects are crucial as workers do not vote only in order to maximise their satisfaction but also to alter the future distribution of types, and through it the future voting outcomes, and the present action (accumulation) taken by the young. It is shown that countries with the same fundamentals may adopt different growth strategies. Countries with lower initial human capital adopt a high wage policy profile that redistributes jobs and income from the young and the capitalists towards prime-aged (insiders) workers. In that case, growth is driven by investment in physical capital. Instead, countries with higher initial human capital adopt a flatter wage policy profile and have a more equal distribution of jobs among workers. For them, human capital drives growth. The physical capital driven growth leads to higher steady state labour productivity but also to lower aggregate income since it is characterised by an inefficient allocation of labour.

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1 Introduction

Since the early 1970s, the contrast in the employment experiences of different OECD countries is striking. Indeed, while a group of E.U. countries faced a continuous rise of unemployment and its persistence at high levels, other OECD countries like the US or the UK experienced a smaller rise in unemployment which came at some point to an end. Moreover, together with the rise in unemployment countries started to diverge in their demographic employment patterns as shown in fig.(1). Stick in to the comparison of the E.U. with the US, it is the lower employment rates (higher unemployment rates) of youth, unskilled adult women, and workers aged 55-64 that explains the main bulk of the differences in aggregate employment and unemployment rates (see Dolado, Felgueroso, and Jimeno, 2001).

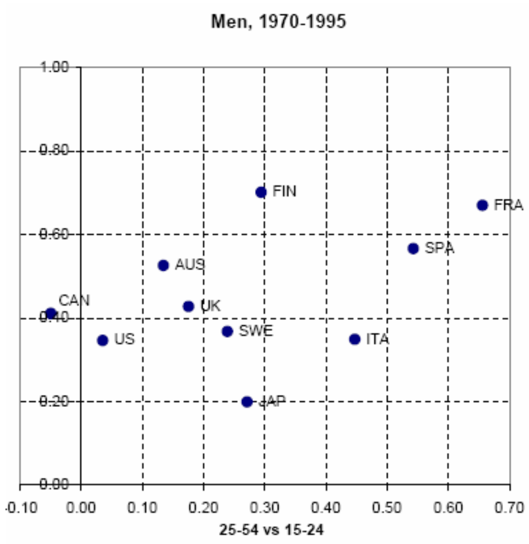

Figure 1: Country-specific changes, across 1970-74 and 1995-96, in the difference of log employment rate (Bertola et al. 2000)

Together with rising unemployment, starting in the late 1970s, hourly productivity in some European countries countries has increased and constantly remained above that of the US[1]. This suggests that while some workers, those that are most likely to

[1]Indeed, some authors document and increasing trade-off between productivity and employment

be unemployed, may suffered from unemployed, others, those that are more likely to rip the gains from higher productivity, may be better-off. However, overall this productivity advantage has been eroded by fewer working hours and lower labor force participation rates for young workers, unskilled adult women, and workers aged 55-64, and consequently does not materialise in higher per capita income and growth performance as suggested in table 1.

Table 1: Decomposition of GDP growth performance (OECD, 1980-98)

	GDP	**Employment**	**Labor productivity**	**Capital deepening**
United States	3.1	2	1.2	0.9
Germany	1.9	0.1	1.9	3.3
France	2	-0.1	2.1	2.3
Italy	2.1	0	2	3.1
Spain	2.3	0.1	2.3	4.3

This preamble suggests that structural factors explaining both the rise and fall of unemployment may exist. A natural way to go along economies structural features, in relation to their labor market outcomes, is to consider their labor market institutions. Indeed, except in a few countries (markedly in the U.K. or Netherlands) sharp changes in labor market institutions have barely been observed, and so till recently. In this paper I propose a common explanation for the rise, the fall and the persistence of unemployment together with its connection to productivity growth and per capita income in terms of endogenous rise, fall and persistence of minimum wage[2]. Hence, I provide a rational for minimum wages legislations that account for economic and political interactions between generations of workers.

Most studies have centered on the short run effect of labor market institutions

(see Beaudry and Collard, 2002) starting in the mid 1970s. But, the european lead in hourly labor productivity end up starting from the mid 1990's in favor of US, except for some northern european countries (see CESifo 2002).

[2]The minimum wage we consider should be interpreted as an institution that redistributes jobs from the "last to the first" workers who entered the labor market.

on employment. However, in so far as those institutions affect youths labor market prospects and lifetime earnings they should also determine their investment choices in human capital, their future labor market prospect and in-fene their political preferences for those institutions. Hence, labor market institutions create political and economic intergenerational linkages which drives the effects of those institutions well beyond the short run. In particular, institutions that effect disproportionately young's labor market prospect should be more critical for growth and productivity than others whose effects are more homogeneously distributed among workers[3].

In this paper I investigate how prime-aged workers (insiders) trade-off today's labor market rigidities against tomorrow's labor market flexibility, and how the outcome of the trade-off shapes the long run educational structure and per capita income of economies. The issue is considered within a three period overlapping generations neoclassical growth model with heterogeneous agents and with an endogenously determined minimum wage. Agents work and acquire human capital[4] while young, they spend their prime-aged period working, and they retire in the third period. I assume that prime-aged worker are less exposed to unemployment than outsiders and own claims on jobs. Hence following the terminology of Saint-Paul (2000) prime-aged workers are political insiders. I further consider that they have a better access to the capital market and hold claims on future firms' physical capital. Hence, labor substitutability between insiders and youths outsiders on one hand, and complementarity between capital and labor, on the other hand, create a trade-off for the insiders' choice of minimum wage. Indeed, being less exposed to unemployment current insiders have a stake on current labor market rigidities which rations equilibrium employment and rises their labor income. However, once they retire they are capitalists (or firms' owners) and they have a stake on labor market flexibility since it increases their capital

[3]The effect of youths unemployment on aggregate human capital can not be exagerated. A critical channel through which unemployment is detrimental for human capital work through its effect on fertility rates and may explain their dramatic drop in countries such as Italy, Spain and Germany.

[4]We often refer in the paper this human capital to education but it may also be thought as on the job training that bears essentailly on workers.

income. In this setting, dynamic aspect of insiders' choices are crucial: when voting over a minimum wage policy, rational and forward looking voters should take into account that they leave the labor market in the next period, and that the next period minimum wage is determined by the choice of next period insiders. Thus, current and future political choices are linked via the impact of current minimum wages on human capital accumulation, which in turn determine future preferences for minimum wages. The minimum wage determines youths training decisions via its impact on youths unemployment rate: a higher minimum wage decreases labor demand for youths and their expected labor income. With credit constraints on human capital investments it increases consumption costs of training and lowers human capital accumulation.

In this context I show that an economy whose insiders adopt a high minimum wage policy profile has a growth based on physical capital accumulation. Minimum wage acts as a tax that redistributes income away from the young (outsiders) and capitalists (retiree) toward savers (insiders) and boosts investment. Instead, an economy relying on a low minimum wage policy has a growth based on human capital accumulation. Indeed, since it relaxes the credit constraint on human capital investment, it fosters youths investments in education and the next period stock of human capital increases. Wether one growth strategy or the other emerges as a political equilibrium depends on the exogenous skill premium. It turns out that the higher is the skill premium the more sensitive is the supply of human capital to youth employment rate. In that case, the low wage policy profile chosen by insiders trigers a human capital driven growth and the economy converges toward full employment. Conversely, for a smaller skill premium, the growth is mainly driven by physical capital accumulation, with a high productivity of labor per effective unit and the existence of unemployment among young workers.

In this paper, the minimum wage is endogenously chosen through repeated voting by rational and forward looking agents. Current policy affects future policies through changes in the average human capital of insiders bring about by the current minimum wage policy. The model builds on the recent work on dynamic political choices in macroeconomics and focus on Markov perfect equilibria, following the work of Krussel *et al.* (1997, 1999). However I depart from these authors and build on the more recent work of Hassler *et al.* (2003) to provide an analytical characterisation of the set of equilibria. In particular, I show that for intermediate values of the skill premium, the economy has multiple steady state political economy equilibria. One of them is characterised by full employment, no minimum wage, and a larger human capital stock; in the other one, youths are partially unemployed due to a positive minimum wage and unskilled make up a larger share of the labour force. Multiple equilibria arise as a consequence of the forward looking behavior of the agents and are due to the strategic political complementarity between today's choice of the minimum wage by current insiders and tomorrow's choice of the minimum wage by tomorrow's insiders. The intuition for the strategic complementarity is the following: if at period t insiders choose to increase the current minimum wage they redistribute income in their favour. Since they are the savers, doing so also increases the stock of physical capital at period $t+1$, once they leave the labor market. As a consequence, next period interest rates are lower due to both the larger amount of physical capital and a smaller amount of human capital. Then, next period insiders have still higher incentives to keep rising minimum wages as their wage gain from doing so is relatively high compared with the cost in terms of forgone future interest rate. This strategy increases their remaining lifetime earning more than the alternative aims at increasing their saving returns. This last strategy necessitates to share employment with young workers. Indeed, with higher labor income young workers human capital investments and the future productivity of physical capital increase. If this strategy was not optimal yesterday it is even less so today. Conversely, I show that for the same set of parameters myopic choices always generate a unique steady state equilibrium. In that case, the economy either converges to a full-employment steady state equilibrium if the skill premium is sufficiently high or to a full-unemployment equilibrium (for young). Hence, the multiplicity of steady state equilibria results from the forward looking behavior of insiders. Two economies having the same structural parameters but with slightly different initial stocks of human capital adopt different labor market policies and converge to steady states with sharp differences in terms of employment, productivity, aggregate output, and average education. In the low employment equilibrium, aggregate labor productivity is higher than in the high employment equilibrium, while due to lower labor force utilisation output per capita is also lower.

I take the stance that minimum wages are endogenously determined. Minimum wages does not bind equally for all workers. If prime-aged workers are relatively insulated against unemployment due to some other labor market institutions or age differentiated firing costs, then youths labor supply is more likely to be rationed by firms. I assume that prime-aged workers are insiders and are protected against unemployment. Indeed, lay-off costs vary according to the seniority of the laid-off workers. The OECD's (1999) employment outlook provides evidence of this differentiated firing costs. Moreover the same study shows that prime-aged men, who are arguably the strongest segment of the labour force, have very similar employment (unemployment) patterns across countries. By contrast, women and above all, young people, appear to be more exposed to the negative effects of labour market restrictions. Other studies show that youth unemployment fluctuations act as buffer stock to macroeconomic shocks and are much more volatile than prime-aged workers unemployment (see Bertola (1999), Jimeno, et *al.* (2001)*).* This is an expected consequence of age-specific firing costs. Considering youths unemployment in their study of OECD countries over the period 1970-1995, Bertola *et al.* (2002) show that in the United States, youths unemployment rose only by 2.2 percentage points for men (and it fell slightly for women), but it increased in Europe by fully 19.2 (males) to 24.5 (females) percentage points. Unemployment for prime-aged men and women also rose in Europe relative to the United States, but by much less than youths unemployment did. Overall, then, rising European unemployment relative to the United States is disproportionately concentrated among youths (and women) over the period 1970-1995. In the paper this age specific unemployment patterns is derived from the design of labor market institutions : labor market protection is not homogeneously distributed among workers, which makes outsiders (the young, women) more likely to be unemployed than insiders (prime-aged workers).

The model's predictions are consistent with empirical studies comparing hourly productivity, employment rates and aggregate output growth performance in the US and a number of European countries, at least til the mid 1990's (see table 1). Notably, Blanchard (1997) documents the rise in capital share in Europe for the period 1980-1995, which does not have its counterpart in the US. The author claims that the change stems from labor demand shifts due to capital biased technical progress and the change in the distribution of rents from workers to firms. Instead, in our model this is due to the different cross country saving patterns that are mediated through the endogenous adoption of labor market institutions that redistribute toward the labor force segment with the highest propensity to save. A related paper by Gordon (1995) documents that the positive trade-off between productivity and unemployment is only temporary and that an adjustment mechanism is at work involving capital accumulation; where after successive periods of productivity slow down, productivity starts recovering and goes hand in hand with employment. In Appendix (B), an extension of the model to endogenous growth is presented which allows to account for this temporary employment productivity trade-off[5]. While initially lower unemployment contributes to higher productivity later on, as the human capital stock increases, physical capital accumulates at higher rates than human capital and productivity starts to be positively associated with the employment rate and human capital accumulation. Neither Blanchard (1997) nor Gordon (1995) consider this joint evolution of human and physical capital, while this is the key mechanism explaining the persistence of unemployment and institutions in my paper.

The paper is clearly related to the literature studying the interaction of labor market institutions and accumulable factors of productions that are essential to growth such as physical and human capital. Hence, it is related to Saint-Paul (1996, 1998), Cahuc and

[5]See also Beaudry and Collard (2002) for evidence on this trade-off.

Michel (1998), Caballero and Hammour (1998), Blanchard (1997) and Gordon (1995). Saint-Paul (1996) characterises multiple equilibria in the human and physical capital space. Higher wage rigidity for unskilled labor creates increasing returns to education and is at the core of the multiplicity. As in this model, equilibria with higher education have lower physical capital than equilibria with lower education. This is a static model where capital is allocated either for human capital accumulation or for final good production. Instead, my model is dynamic and the multiplicity is due neither to increasing returns nor to an externality but to the dynamic political and economic feedback driven by the forward looking behavior of agents. However, the equilibria have the same characteristics with respect to the capital labor ratio. Endogeneity of labor market institutions and growth is tackled in Saint-Paul (1998), who analyses political support for employment protection. A noteworthy conclusion of this paper that relates to mine is that initial support for labor rigidity generates its own future constituency by maintaining a larger labor force in old vintages technologies. In my paper workers poorly endowed with human capital favor wage policies that generate future support for more rigidity[6], which deters human capital investment and prepares the ground for the future support of rigidity. The impact of youths unemployment on education differs from that assumed by Cahuc and Michel (1996). In their paper higher unemployment rises returns to skill and triggers investments in education, in my model owing to credit constraints, unemployment rises educational costs and slow down human capital accumulation. Consequently, even if one introduces capital externality in production as they do (as in Appendix B), equilibria with unemployment still display lower steady state growth and lower human capital. I also obtain multiple steady state equilibria with sharp differences in human capital intensity, but this is due to the consequence of self-reinforcing intergenerational uncoordinated behaviors of insiders holding claims on future physical capital. The paper shares the general concern of Blanchard (1997)

[6]Introducing human capital investment, Saint-Paul (1998) multiplicity result stems from a positive feedback between specific human capital investment (firm level) and support for rigidity. The distinction regarding human capital is qualitative: economies with low employment protection invest less in specific human capital. Hassler *et al.* (1998) provide a similar political economy argument that relates to the specificity of human capital.

and Caballero and Hammour (1998) as it shows that institutional factors are as relevant for the long run (growth) as they are for the short run (unemployment).

The rest of the paper is organised as follows. The next section presents the general framework of the model. The third section derives the equilibrium economic and policy rules and contains the main results of the paper. The last section concludes. An appendix contains the proofs and an extension of the model to endogenous growth.

2 Minimum wage in the neo-classical growth model with heterogeneous agents

The economy operates in a perfectly competitive environment and economic activity extends over infinite discrete time. I consider a three period overlapping generations structure, where each generation has a measure one of individuals. Individuals are non altruistic. In each period, firms hire labor from young and prime-aged workers and capital from retirees to produce a unique final good. The number of efficiency units of educated and uneducated labor in each period is determined by the educational choices of young and primed aged generations of workers.

2.1 Firms

A large number of firms acting competitively on goods and factors markets produce a unique final good. The production technology is described by a CRS production function F with physical capital K and human capital (efficiency units) H as inputs:

$$\begin{aligned} Y_t &= F(K_t, H_t) = A(K_t)^{\alpha} H_t^{1-\alpha} \qquad (1) \\ \text{with } H_t &= L^o(\tilde{l}_t) + L^i(E_t) = H(\tilde{l}_t, E_t) \end{aligned}$$

The production of effective labor H is additive with prime-aged insiders and young outsiders human capital as arguments, $L^i(E_t)$ and $L^o(\tilde{l}_t)$. Each educated (uneducated) insider provides η_e (η_u) efficiency units and $\eta_e > \eta_u$. Hence if we note E_t the mass of educated insider in t and since each generation is of measure one, insiders human capital supply is $L^i(E_t) = \eta_e E_t + (1 - E_t)\eta_u$. Assuming that outsiders labor is worth one efficiency unit and denoting by $\tilde{l}_t$, the youths employment rate, outsiders provide then $L^o(\tilde{l}_t) = \tilde{l}_t$ efficiency units of labor. The production function can then be written:

$$Y = A(K_t)^{\alpha}(\tilde{l}_t + \eta_u + E_t \Delta\eta)^{1-\alpha}$$

where $\Delta\eta = \eta_e - \eta_u > 0$ is a measure of the productivity gap between both types of insiders. Hence, depending on youths employment rate, at every period t, $H_t \in [L^i(E_t), L^i + 1] = [\underline{H}_t(E_t), \underline{H}_t(E_t) + 1]$, the wage per effective labor unit is bounded from above for any K_t by $F_H(K_t, L^i_t(E_t))$.

Under the assumption of full depreciation of capital, profit maximising firms demand capital and effective labor according to the equilibrium conditions on prices and quantities:

$$\begin{aligned} w &= F_H(K, H) \\ \text{and } R &= F_K(K, H) \end{aligned} \qquad (2)$$

Indeed, factor prices are fully characterised by the capital labor intensity $k(\tilde{l}_t; K_t, E_t) = \frac{K_t}{H(\tilde{l}_t; E_t)}$ and we can define the inverse demand function for labor, $w(k)$ with $w' > 0$, and for capital $R(k)$ with $R' < 0$.

2.2 Individuals

Preferences

The population has an overlapping generation structure. The economy is populated by a continuum of individuals of mass one. Each individual lives for three periods and earns labor income in the two first periods of his life. At birth, knowing their idiosyncratic educational costs, agents decide wether to acquire education or not. Young agents have no access to the capital market neither for borrowing nor for storage, which implies that human capital investment[7] has to be borne against their first period con-

[7] We will often refer in the paper to this human capital investment as education. But it may also be thought of as on the job training paid by the workers out of their first period income.

sumption[8]. During this first period it is assumed for simplicity that both skilled and unskilled agents have the same productivity. One may assume for instance that learning by doing while not in school allows uneducated agents to catch up with the educated agents productivity so that initially both agents have the same productivity. However, educated workers have higher second period productivity. At the beginning of her second period of life, each agent gives birth to an off-spring, and hence the population is constant. The second period agents become insiders with certainty independently of their previous labor market experience[9]. During this second period, insiders vote over a minimum wage for this period, consume and save for their old age. Entering the third period, agents leave the labor market, loose their insider status and its associated right to vote on labor market institutions, and consume the proceeds of their savings. Agents preferences are represented by a separable utility function separable in each period consumption. The preferences of the generation born in a generic period $t-1$ are represented by a logarithmic utility function[10]:

$$U(c_{t-1}^{o}, c_{t}^{i}, c_{t+1}^{r}) = \ln(c_{t-1}^{o}) + \beta \ln(c_{t}^{i}) + \beta^{2} \ln\left(c_{t+1}^{r}\right),$$

where β is the discount factor and the subscript refers to the timing of consumption and the upper-scripts stand respectively for young (outsider), prime-aged (insider) and old (retiree).

To concentrate on insiders saving behavior I further assume that youth consume their whole income. Each young worker is endowed with one unit of time. I assume that unemployment spells are uniformly distributed among young workers[11]. Hence if

[8]However, we can not simply assume that they have an infinite discount rate since in that case their educational choice will be independent of the skill premium and thus of the minimum wage through its impact on the employment rate.

[9]So that is a particular definition of ”insiders”, which is not the same as in the dynamic insider-outsider theory.

[10]The log-linear utility function is chosen for computational convenience, the crucial assumption being that the utility function is homothetic in the second and third period consumptions. With this hypothesis, a constant fraction of the second period income is saved for the third period consumption.

[11]Rather than assuming job rationning, we thus implicitely assume that the is rationning in the quantity of labor provided at each job due to the minimum wage. In this way, we do not need to introduce an insurance or unemployment benefit scheme, whose absence would lead to zero consumption.

$\tilde{l}_t$ is the youth employment rate, firms hire a fraction $\tilde{l}_t$ of each young individual unit of time endowment which is worth one efficiency unit. Hence the young first period income is simply $\tilde{l}_t w_t$.

The insider (or prime-aged worker) income, y_t^i, is divided between second period consumption and saving for his retirement consumption. An insider solves the following program:

$$\begin{aligned} & \max_{s_t} \ln c_t^i + \beta \ln c_{t+1}^r \\ \text{s.t. } s_t + c_t^i &= y_t^i \\ c_{t+1}^r &= s_t R_{t+1} \\ c_t^i &\geq 0,\ c_{t+1}^r \geq 0 \end{aligned}$$

With logarithmic preferences, the saving decisions of insiders are independent of their saving's returns and each insiders saves a fraction $\frac{\beta}{1+\beta}$ of his labor income for his retirement. The insiders indirect utility function writes:

$$V(y_t^i; R_{t+1}) = (1+\beta)\ln y_t^i + \beta \ln R_{t+1} + c \tag{3}$$

where $c = \ln(1+\beta)^{(1+\beta)}\beta^{\beta}$ is a constant.

Youths educational choices

Knowing their educational cost σ_i, agents decide whether to acquire education or not. I assume that σ_i is iid within and between generations, in particular it is not linked to any parental characteristic, σ_i follows a P.D.F. f and C.D.F. Φ. The total cost of education at time t is:

$$\sigma_i W_t$$

This cost is equal to the average wage W_t times an individual specific cost σ_i, and is paid by the young out of his first period income[12]. Education or training raises second period productivity as $\eta_e > \eta_u$. Labor supply is assumed to be inelastic and in $t+1$ an educated (resp. uneducated) agent earns $\eta_e W_{t+1}$ (resp.$\eta_u W_{t+1}$). A young born at t choosing to acquire education has the indirect utility function:

$$V_e^1 = \ln\left(\tilde{l}_t W_t - \sigma_i W_t\right) + \beta V(\eta_e W_{t+1}; R_{t+2}) \tag{4}$$

while the unskilled indirect utility is:

$$V_u^1 = \ln\left(\tilde{l}_t W_t\right) + \beta V(\eta_u W_{t+1}; R_{t+2})$$

Hence, while both types of agents receive the same average income when they are young, once they get older type e productivity increases more than type u productivity. Acquiring education is costly but it increases second period productivity at a higher rate. A young will invest in education if and only if the utility derived from doing so is higher or equal to the utility derived from not investing in human capital,

$$V_e^1 \geq V_u^1,$$

$\Leftrightarrow$

$$\log\left(1 - \frac{\sigma_i}{\tilde{l}}\right) + \beta(1+\beta)\log\frac{\eta_e}{\eta_u} > 0$$

This condition determines a critical education cost function, $\sigma_t^*(\tilde{l}_t)$, such that only those individuals with an education cost σ_i below this threshold choose to invest in

[12]With imperfect markets, futur labor income cannot serve as collateral as argued by Ljungvist (1993).

I have not specifically introduced an educational sector. One may assume that skilled insiders are input in the education production sector which pay a competitive wage. Then, σ_i is the time a youngster has to buy from an insider to be educated. In this case one has to take out from the insiders labor supply to firms the time spent teaching the youths and add a financing scheme. To simplify, we just assume an exogenous consumption cost of acquiring general education which should be paid by the end of the first period due to capital market imperfections.

human capital:

$$\sigma_t^*(\tilde{l}_t) = \tilde{l}_t \left[1 - \left(\frac{\eta_u}{\eta_e} \right)^{\beta(1+\beta)} \right] \tag{5}$$

$\sigma_t^*(l_t)$ increases with the employment rate l_t. This is due to the fact that with credit constraints on human capital investments, the marginal cost of education decreases when youths employment opportunities rise. Better employment opportunities exert a positive wealth effect that lowers the utility cost of education. A higher wage gap serves as an incentive for investing in education since $\frac{\partial \sigma_t^*}{\partial \frac{\eta_e}{\eta_u}} > 0$. Those who attend school while young are educated insiders next period, their mass is $E_{t+1} = \int_0^{\sigma_t^*(\tilde{l}_t)} f(\sigma_i) d\sigma_i$. I assume that σ_i follows a uniform distribution over the domain $[0, \bar{\sigma}]$. The supply of educated at $t+1$ is then:

$$E_{t+1} = E(\tilde{l}_t) = \frac{\tilde{l}_t \left[1 - \left(\frac{\eta_u}{\eta e} \right)^{\beta(1+\beta)} \right]}{\bar{\sigma}} = x\tilde{l}_t \tag{6}$$

I refer to the function $E(.)$ as the enrollment function. The cost of acquiring education is indexed by $\bar{\sigma}$, a higher $\bar{\sigma}$ means that acquiring education is more costly and consequently educational choices are less responsive to employment opportunities. The next period stock of educated E_{t+1}, depends on the current youth employment rate, $\tilde{l}_t$.

Physical capital supply

From the assumption that young outsiders have no access to the capital market, physical capital at period t, K_t, is equal to the savings of period $t-1$ insiders S_{t-1}, i.e.,

$$S_{t-1} = K_t \ \forall t \tag{7}$$

and since the unique savers are the prime-aged workers, the following market clearing equality is verified:

$$\frac{\beta}{1+\beta} L^i(E_t) W_{t-1} = K_t \tag{8}$$

from which we derive the dynamics of capital stock:

$$K_{t+1} = \frac{\beta}{1+\beta} L^i(E_t) A(1-\alpha) \left(\frac{K_t}{H(\tilde{l}_t; E_t)} \right)^{\alpha}, \tag{9}$$

the capital stock dynamics depend on the evolution of E_t and of the youth employment rate.

3 The political-economic equilibrium

3.1 The insiders objective function

I specify now the policy choice. The state of the economy at t is summarised by a stock of capital K_t, and the insiders stock of human capital E_t. With the assumption that insider status prevents prime-aged workers from being unemployed and with the assumption of substitutability between both types of labor, the choice of minimum wage is equivalent to a choice over youths employment rate $\tilde{l}_t$. Despite their income heterogeneity, prime-aged workers share the same objective regarding the minimum wage policy. This is due to the fact that the insider status applies to all prime age workers irrespective of their education type and that with logarithmic preferences the propensity to save is independent of income[13]. Hence, I can refer to a representative insider or a union seeking a minimum wage policy that maximises the insiders lifetime utility.

Lemma 1 *The representative insider objective function is:*

$$V(\tilde{l}_t, \tilde{l}_{t+1}; E_t, E_{t+1}) = -S \ln H_t(\tilde{l}_t; E_t) + \ln H_{t+1}(E(\tilde{l}_t), \tilde{l}_{t+1}) \tag{10}$$

with $S = \frac{\alpha}{1-\alpha}\left(\frac{1}{\beta} + \alpha\right)$

Proof. Plug the capital dynamics equation (9) and the equilibrium price conditions (2) in the insiders indirect utility function (3). Keeping only terms that depend on policy variables $\tilde{l}_t$ and $\tilde{l}_{t+1}$ gives (10). ■

[13] With more general preferences or production function, majority rules among insiders choice should be applied. This would rule out any feasible analytical determination of the equilibrium policy and thus one should turn to numerical solutions of the type used in Krussel et al. (1996, 1999).

According to lemma (1) the representative insider objective function is independent of the physical capital stock. The unique relevant state variable in the optimal minimum wage policy choice of insiders is the current human capital stock E_t. The parameter S can be interpreted as the intertemporal elasticity of substitution of human capital. Namely, an insider will accept to trade one unit of present human capital in exchange of S units of future human capital. Naturally, this price of human capital substitution rises as the agents become more impatient (lower β) and as capital becomes more productive (high α), since in that case the value of current savings is higher. Tomorrow's stock of human capital depends on today's chosen minimum wage via the impact of employment opportunities on the human capital investment of the young. This objective function shows the main trade-off faced by current insiders: as wage laborer they rather benefit from a lower $\tilde{l}_t$ (labor market rigidity) to minimize today's aggregate human capital H_t conditional on keeping their job, but as tomorrow's capitalist they will rather benefit from a lower minimum wage (labor market flexibility) to foster human capital accumulation H_{t+1} so as to increase the returns of their savings.

I shall stress the need to adopt a particular equilibrium criterion that restricts the set of possible equilibria. In the unrestricted case, rational and forward looking insiders would play an intergenerational game with an infinite sequence of players and solve the following nested program:

$$\begin{aligned} & \max_{\tilde{l}_t} - S\ln H_t(\tilde{l}_t; E_t) + \ln H_{t+1}(E_{t+1}, \tilde{l}_{t+1}) \qquad (11) \\ s.t.\ E_{t+1} &= E(\tilde{l}_t),\ \text{and } \tilde{l}_t \in [0,1] \text{ and } \tilde{l}_{t+1} \in [0,1] \\ s.t.\ l_{t+1} &= \arg\max -S\ln H_{t+1}(\tilde{l}_{t+1}; E_{t+1}) + \ln H_{t+2}(E_{t+2}, \tilde{l}_{t+2}) \\ s.t.\ E_{t+2} &= E(\tilde{l}_{t+1}),\ \text{and } \tilde{l}_{t+1} \in [0,1] \text{ and } \tilde{l}_{t+2} \in [0,1] \\ & ... \end{aligned}$$

An equilibrium will then be a sequence of policy functions $\tilde{l}^*_t(.)$ such that $\tilde{l}_t = \tilde{l}^*_t(.)$ solve (11) given the optimal sequence of choices of past and future generations of insiders $\{\tilde{l}^*_0(.), \tilde{l}^*_1(.), ..., \tilde{l}^*_{t-1}(.), \tilde{l}^*_{t+1}(.), ...\}$. The political equilibrium so obtained is a perfect Nash equilibrium where each middle-aged generation chooses the optimal minimum wage by fully discounting the effect it has on next period sequence of insiders choices. Models incorporating repeated voting with possible strategic interactions can usually only be solved numerically. Notably, in the context of redistributive taxation, this is the path followed by Krusell, *et al.* (1996), Krusell and Rios-Rull (1996, 1999) and more recently by Saint-Paul (2001).

Instead, I shall first restrict to a subset of sub-game perfect equilibria namely the set of Markov perfect equilibria and rely on the methodological approach put forward by Hassler, *et al.* (2003) to provide an analytical characterisation for the set of equilibria. To my knowledge their approach has not been applied to the study of the dynamics of labor market institutions with their connections to the accumulation of physical and human capital, which are essential to growth. The next definition adpated from Hassler, *et al.* (2003) specifies the equilibrium concept adopted:

Definition 1 *A Markov perfect political-economic equilibrium is:*
A constant policy rule that for the current state of the economy gives the policy chosen by the critical voter, $\tilde{L} : [0,1] \to [0,1]$
A private decision rule for human capital supply, $E : [0,1] \to [0,1]$
Such that the following functional equations hold

$$\tilde{L}(E_t) = \underset{\{\tilde{l}_t\}}{\arg\max} V(\tilde{l}_t, \tilde{l}_{t+1}, E_t, E_{t+1})$$

$$s.t.$$

$$\begin{aligned} \tilde{l}_{t+1} &= \tilde{L}(E(\tilde{l}_t)) \\ E_{t+1} &= E(\tilde{l}_t) = x\tilde{l}_t \\ \tilde{l}_t &= \tilde{L}(E(\tilde{l}_{t-1})) \end{aligned}$$

and a sequence of prices for capital and labor such that economic equilibrium relation on prices (Eq. 2) and on quantities (Eq.7) hold $\forall t = 0, ..., \infty$

It follows that in a Markov equilibrium the set of utility-maximising policies depends only on the current state of the economy through a constant mapping. Hence, the constant policy rule implies that for any s if $\tilde{l}_{t+s} = \tilde{L}(E(\tilde{l}_{t+s-1}))$ then $\tilde{l}_t = \tilde{L}(E(\tilde{l}_{t-1}))$. Since only current insiders vote over the minimum wage and since they all have the same objective functions, the critical voter is a representative insider. In particular, there is no ex-post conflict between insiders since they all have the same investment opportunities[14] there is no room for strategic votes that will change the identity of the critical voter. Strategic voting is present in so far as current insiders are aware of, and integrate in their own policy choice, the influence they have on future policies that will be adopted. The optimal choice of minimum wage is worked-out in a general equilibrium context where all prices are endogenous and voters are forward looking and aware of the consequences of their policy on the future policy choices.

The rationality and forward looking assumptions imply that insiders take into account that the chosen minimum wage has an impact on tomorrow's preferred minimum wage via the impact of today's minimum wage on tomorrow's state variable, that is $E_{t+1} = E(\tilde{l}_t)$. Hence, while myopic insiders do not consider strategic interactions, that is $\frac{\partial \tilde{l}_{t+1}}{\partial \tilde{l}_t} = 0$, rational and forward looking insiders do. I show in Proposition (3) that a steady state political-economic equilibrium with myopic voters is always a corner solution with either full employment or youths being all unemployed. Instead, successive choices of forward looking insiders may lead to full employment with no minimum wage or to an interior solution with youth being partly unemployed. Since there is no conflict among prime-aged workers, the insiders' optimal choice of minimum wage policy boils down to that of a representative union whose objective is to maximise the remaining lifetime utility of its current members[15].

[14]See Saint-Paul and Verdier (1997), where agents have different investment opportunities such that some agents can free-ride on the effort of others by investing abroad.

[15]We may assume that at the time the vote takes place young workers have not yet entered the labor market. Since the three generations are of the same size it suffices then to assume that senior workers are just slightlty more powerfull than capitalists to decide about labor market institutions.

I posit that the critical voter's is always an insider. Assume instead that the critical voter is not an insider. With a constant population the three generations have the same size. The capitalists or retirees unambiguously vote for full employment so as to maximise their saving's returns. The choice of young is not that clear. Since they do not have access to capital market, they benefit from the minimum wage since incomes transferred to insiders allow them to work with more capital tomorrow. This effect of financial repression on accumulation is in the spirit Japelli and Pagano (1994) who developed a model where constraints on young borrowing increase the steady state utility of all agents in the economy by fostering capital accumulation. I should point out that with generational differences in access to capital market, higher minimum wages do not necessarily reduce youths welfare. Indeed, an equilibrium featuring minimum wages and youth unemployment may even Pareto dominates an equilibrium with full employment and no minimum wage. I next analyse the consequences of the forward looking behavior of insiders on the choice of minimum wage.

3.2 The equilibrium minimum wage policy

I shall further restrict production and preference parameters such that interior solutions are possible equilibria outcome. We denote $z = \frac{dE}{dl} * (\eta_e - \eta_u) = x\Delta\eta$, the net increment in future human capital brought by a marginal increase in youths employment rate. I assume that this value is bounded as follows: $(S-1) < z \leq (S-1)\frac{(1+\eta_u)}{\eta_u-(S-1)}$ and that $S-1 < \eta_u < (S-1)(S+1)$. I derive in the appendix the equilibria for the full set of possible parameters values. The next proposition characterises the equilibrium policy rule for this restricted set of parameters and the corresponding human capital dynamics.

Proposition 1 *The equilibrium policy rule followed by insiders is:*

$$\tilde{L}(E_t) = \tilde{l}_t = \begin{cases} aE_t + b & \textit{if} \quad E_t < \frac{1}{a}\left(\frac{1-b}{ax} - b\right) \\ \frac{1-b}{ax} & \textit{if} \quad \frac{1}{a}\left(\frac{1-b}{ax} - b\right) < E_t < \frac{1}{a}\left(\frac{1-b}{ax} - b'\right) \\ aE_t + b' & \textit{if} \quad \frac{1}{a}\left(\frac{1-b}{ax} - b'\right) < E_t < \frac{1-b'}{a} \\ 1 & \textit{if} \quad E_t > \frac{1-b'}{a} \end{cases} \tag{12}$$

The private decision rule follows the dynamics,

$$E_{t+1} = E\left(\tilde{L}(E_t)\right) = \begin{cases} ax\left(E_t - E^*\right) + E^* + dx & \text{if} \quad E_t < \frac{1}{a}\left(\frac{1-b}{ax} - b\right) \\ \frac{1-b}{a} & \text{if} \quad \frac{1}{a}\left(\frac{1-b}{ax} - b\right) < E_t < \frac{1}{a}\left(\frac{1-b}{ax} - b'\right) \\ ax\left(E_t - E^*\right) + E^* & \text{if} \quad \frac{1}{a}\left(\frac{1-b}{ax} - b'\right) < E_t < \frac{1-b'}{a} \\ x & \text{if} \quad E_t > \frac{1-b'}{a} \end{cases}$$

with $a = \frac{\Delta\eta}{S-1}$, $b = \frac{\eta_u(z-(S-1))}{(S-1)(z+1)} > 0$, $b' = \frac{z\eta_u - S(1+\eta_u)}{z(S-1)}$, $d = \frac{S}{S-1}\frac{1+\eta_u - z}{z(z-1)}$ *and* $E^* = \frac{b'x}{1-ax}$

Proof. See appendix. ■

The policy rule is piece wise linear with respect to the state variable E_t: it has a constant slope and a changing intercept. The dynamics for minimum wage policy implies that the economy may display either a unique or multiple steady state equilibria. When the steady state is unique the economy converges either to a full employment equilibrium or to an equilibrium with youths being partly unemployed. Full unemployment is never an equilibrium. When the economy has multiple steady states equilibria two of this equilibria are stable and one is unstable. The next proposition states the possible equilibria configuration that arises as a function of the net increment in human capital brought by a marginal increase in youths employment rate (z) and characterises the two possible steady state political-economy equilibria in terms of employment[16].

Proposition 2 $\forall S > 1$ *then there is a value* $z_h \in ((S-1), c(S-1))$ *with* $c = \frac{1+\eta_u}{1+\eta_u - S} > 1$ *such that:*

- *if* $z < S$ *the economy converges to a unique equilibrium with youth unemployment (see fig. 3).*
- *if* $z > z_h$ *the economy converges to a unique equilibrium with full employment (see fig. 2)*

[16] See the appendix for equilibria corresponding to all possible parameters values.

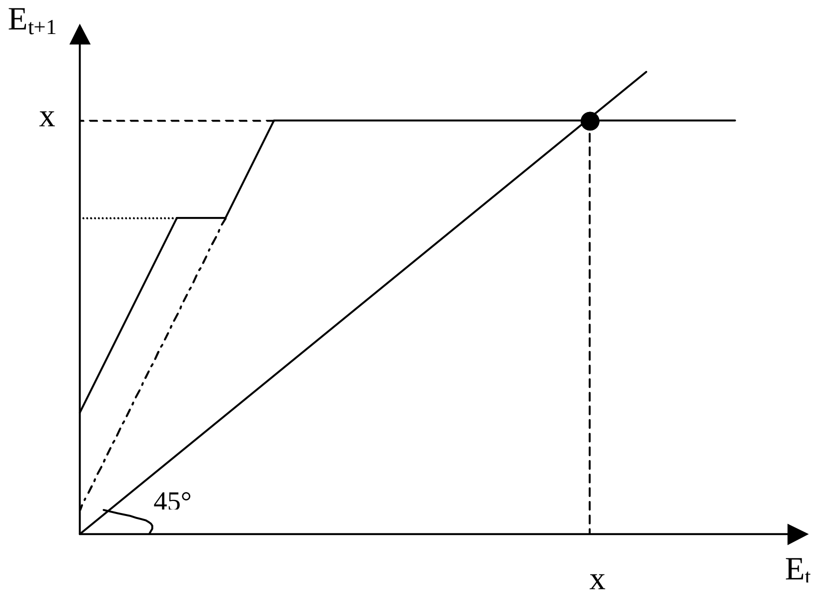

Figure 2: High employment steady state equilibrium: $z > z_h$

- *if* $S < z < z_h$ *the economy has multiple equilibria, an unstable equilibrium and two stable equilibria, one with strictly positive youth unemployment and the other with full employment (see fig. 4).*

The core results of propositions (1 and 2) lies on the political strategic complementarity between successive generations of insiders' choices. Namely, a current increase in minimum wages (unemployment) prepares the ground for future increases, while a current decrease prepares the ground for later decreases, thus creating a positive feedback between successive policy choices mediated through equilibrium prices on the capital and labor market. To grasp the intuition for this positive strategic complementarity let's consider the standard inverse labor demand schedule depicted in fig. (5). The wage schedule is a decreasing function of labor whose slope is an increasing function of current stock of capital: $\frac{\partial W_t}{\partial L_t \partial K_t} < 0$ and $\frac{\partial^2 W_t}{\partial^2 L_t} > 0$. Assume that current insiders decide to increase the minimum wage, the marginal drop in youths employment rate rises insiders' wage by $\overline{AB}$ in fig.(5). However, the rise in minimum wage increases the next period stock of physical capital and because of its adverse effect on youth

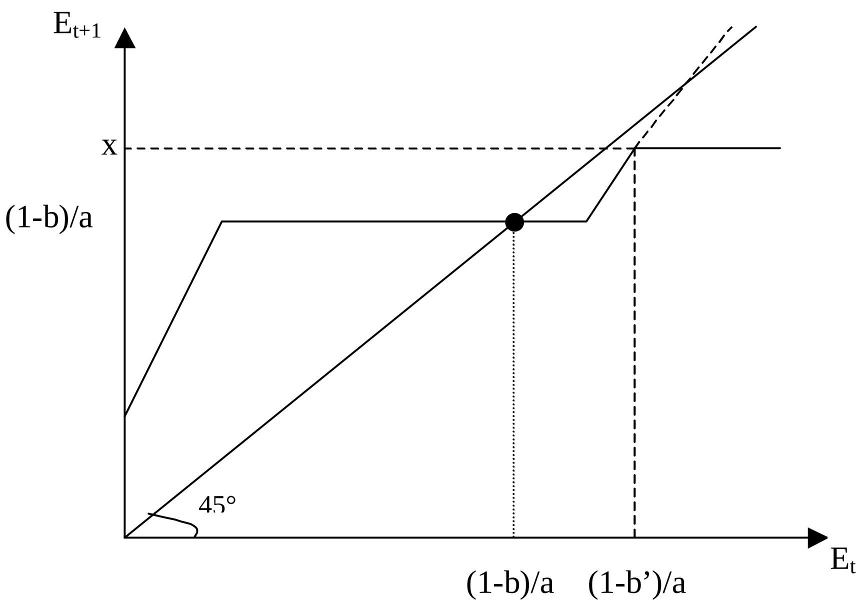

Figure 3: Unemployment steady state equilibrium: $z < S$

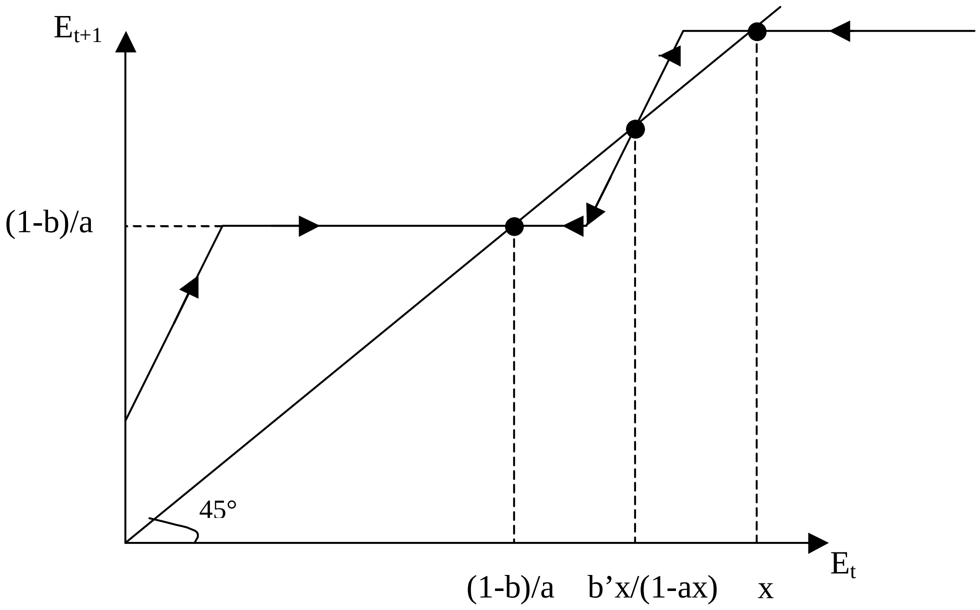

Figure 4: Multiple steady states equilibria: $S < z < z_h$

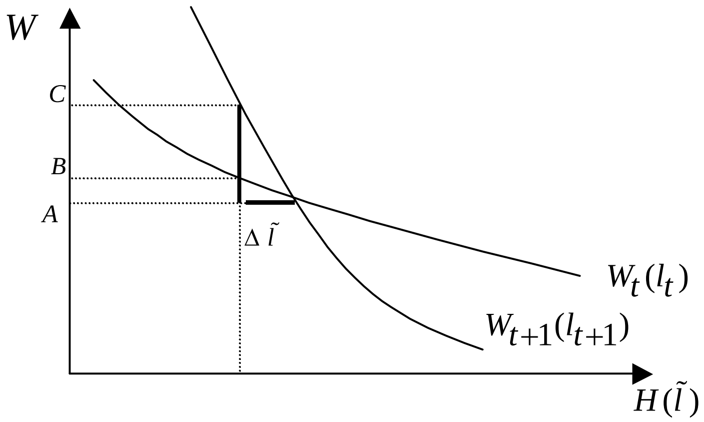

Figure 5: Wage profile following an increase in the minimum wage.

employment rate it raises the utility cost of investing in human capital which lowers the next period human capital endowment of insiders. Hence the period $t+1$ wage schedule, $W_{t+1}(l_{t+1})$, is steeper than the current wage schedule meaning that for the same marginal increase in the minimum wage, the wage gain for period $t+1$ insider is still higher ($\overline{AC} > \overline{AB}$). Hence, if it was optimal for period t insiders to rise minimum wages it is even more so for period's $t+1$ insiders. Assume instead that period t insiders decide to decrease the minimum wage (see fig.6) such that youths employment rate rises. The lower minimum wage decreases current insiders income by $\overline{A'C'}$ and consequently the next period's stock of physical capital is lower, while the better labor market prospect for youths fosters their investments in human capital. These combined effects make the $t+1$ wage schedule still flatter. Hence, if it was optimal for the period t insiders to decrease minimum wages it is even more so for the period $t+1$ insiders since the wage cost from a marginal decrease are lower ($\overline{A'B'} < \overline{A'C'}$). By similar arguments, it can be shown in the (R_{t+1}, L_{t+1}) space that a lower youths employment rate decreases the interest cost of higher physical capital accumulation $\frac{\partial R_{t+1}}{\partial K_{t+1} \partial L_t} < 0$, and makes the adoption of high minimum wage policy more likely.

Considering Eq. (3) a higher youth unemployment has a direct and an indirect negative price effect, and a positive income effect on insiders utility. The price effect works through the direct negative impact of higher physical capital accumulation on

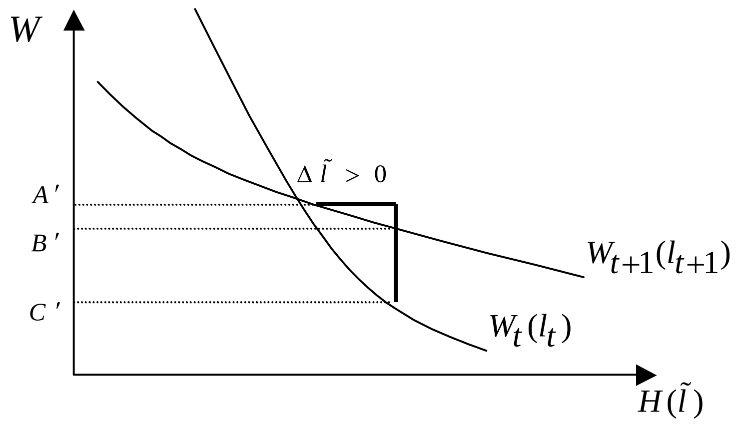

Figure 6: Wage profile following a decrease in the minimum wage.

future interest rates. A higher minimum wage has also an indirect negative effect: a lower youth employment rate increases educational costs and lowers human capital accumulation and next period returns on savings. The positive income effect is due to the substitutability between insiders and outsiders labor.

If the sensitivity of human capital investments to youth employment rate, z, is low ($z < S$), the indirect negative price effect is low, and the positive income effect overcome the direct price effect. Thus, for all possible states of the economy insiders prefer to indirectly tax the outsiders, the gain in current income overcome the cost of lower future returns on savings.

Instead for a high responsiveness of human capital investments to youths employment rate ($z > z_h$), insiders optimal strategy for increasing their remaining lifetime utility is to rise their savings returns. In that case insiders care about the adverse effect that youth unemployment has on human capital accumulation. The lower minimum wage increases current youths employment rates, decreases marginal educational costs and fosters human capital accumulation. This strategy is adopted for all states of the economy if z is sufficiently high, since in that case the indirect negative price effect always overcome the positive income effect.

Hence two growth strategies emerge. One strategy is directed toward physical capital accumulation with a minimum wage that act as a tax on youths and retiree income and redistribute toward savers. The other strategy is instead based on human capital accumulation with a higher rate of investment in human capital. Indeed, one can check that investment as share of GDP, $\frac{I}{Y}$, rises when minimum wages rises (that is when there are human capital depletion):

$$\frac{I}{Y} = (1-\alpha)\,\frac{L^i(E_t)\frac{\beta}{1+\beta}W(k(E))}{LW(k(E))} = (1-\alpha)\,\frac{\beta}{1+\beta}\frac{L^i(E_t)}{L(\tilde{l}_t(E_t), E_t)}$$

and

$$\frac{\partial(I/Y)}{\partial(E)} < 0$$

For intermediate values of z there exists an initial human capital endowment for insiders such that they are indifferent between one or the other strategy and the economy has multiple steady state equilibria. Starting from this initial stock, if one generation starts to increase the minimum wage so will do the next generations til the economy reaches a positive unemployment trap. The result is due to the forward looking behavior of insiders, the intuition is as follows (see fig. 4): starting from E^* an increase in youth employment rate (increase in $\tilde{l}_t$) decreases insiders current income, however R_{t+1} increases both because next period stock of capital K_{t+1} is lower and because next period human capital is higher due to human capital accumulation and the complementarity between human and physical capital. This effect alone is not sufficient to generate multiple equilibria, if on top of that insiders rationally expect their vote and the next period insider's vote to be strategic complement $\frac{\partial l_{t+1}}{\partial l_t} > 0$, then they have still more incentives to increase l_t, because the price effect (higher R_{t+1}) overcome the negative income effect (lower W_t). The economy starts then a growth strategy based on human capital accumulation and converges to a steady state equilibrium with full employment. Instead, starting from E^*, a higher minimum wage increases current insiders income and decreases future interest rate since the economy will be endowed with more physical capital and less human for using it. While this effect alone does not generate the multiplicity of equilibria adding the rational expectation of strategic policy complementarity, R_{t+1} decreases further. The economy starts then a growth strategy based on physical capital accumulation and converges to a steady state equilibrium with young unemployment. It is worth to note that when multiple steady state equilibria exists the economy has a low employment trap to which corresponds a specific growth equilibrium path. The political unemployment trap corresponds to $\frac{1}{a}\left(\frac{1-b}{ax}-b\right) < E_t < \frac{1}{a}\left(\frac{1-b}{ax}-b'\right)$. For this range of relatively low values of insiders' educational stock whatever are the next period rational employment rate expectations, the current insiders' endowment of human capital is so low that the wage gain from choosing a low employment rate overcomes the interest rate loss. In turn the chosen low employment rate depresses human capital accumulation and prepares the ground for the next generations of insiders to keep on choosing low employment rate (see the proof of proposition 1).

To gain insights on the relevance of considering the forward looking behavior of policy choices, I consider next the optimal policy adopted by myopic rational insiders. Insiders are myopic in as much that they do not consider the influence they have on future policy preferences. Namely, myopic insiders consider $\tilde{l}_{t+1}$ as a parameter, $\tilde{l}^{my}_{t+1} = \bar{l}^a_{t+1}$ with $\frac{\partial \tilde{l}^{my}_{t+1}}{\partial \tilde{l}_t} = 0$, an interior policy choice for myopic insiders expected the employment rate $\bar{l}_{t+1}$ to prevail the next period is:

$$\tilde{l}^{my}_t(E_t, \bar{l}_{t+1}) = \frac{\Delta\eta}{S-1}E_t + \frac{\eta_e\,(z-S) - S\bar{l}^a_{t+1}}{(S-1)\,z}$$

where $\bar{l}^a_{t+1}$ is the period $t+1$ rational expectations on future policy. The rationality of expectations imposes that expectations should be monotonic[17], that is if $\tilde{l}_t > \bar{l}^a_{t+1} = l^{my}_{t+1}$ then $\tilde{l}^{my}_{t+1} > \bar{l}^a_{t+2} = l_{t+2}$, while if $\tilde{l}^{my}_t < \bar{l}^a_{t+1} = l_{t+1}$ then $\tilde{l}_{t+1} < \bar{l}^a_{t+2} = \tilde{l}^{my}_{t+1}$. The intuition is the same as the one that we have grasped in figs. (6) and (5). The next proposition characterises the myopic steady state equilibrium outcome:

Proposition 3

[17]The details are provided in the Appendix in the proof of proposition (3)

- *The steady state Myopic Equilibrium policy is:*

$$\tilde{l}^{my} = \begin{cases} 1 & \textit{if} \quad z \quad > \quad S \\ 0 & \textit{if} \quad z \quad \leq \quad S \end{cases} \tag{13}$$

Intuitively if the gains from higher human capital accumulation brought by a marginal increase in current employment rate (z) outweighed the quantity at which insiders are indifferent between current and future increases (S), then the successive choices of insiders will drive the economy to full employment. The converse hold for $z < S$. The discrepancy with the forward looking (FL) decisions is due to the fact that FL insiders can free ride on the decisions taken by the next period insiders when they decide to vote for a decreasing minimum wage profile. On one side an insiders wishing to increase youths employment rate find greater benefit from doing so if they expect next period's insiders to react by increasing their own period's employment rate (for the same wage loss future interest rates gains are higher). This explains the non linearity of the minimum wage policy. Myopic insiders can not rely on this complementarity of behavior. Accordingly myopic behavior should lead to slower convergence to full employment than forward looking behavior does. On the other side FL insiders that choose a high minimum wage profile bears part of the cost of the behavior of future generations of insiders. Indeed since those insiders will react by increasing their own period's minimum wage, for the same positive income effect (wage gain) insiders have to incur a higher negative price effect (interest rates loss). Consequently the forward looking insiders minimum wage policy is in some sense "more sluggish" downward than it is upward. This explains that there exist steady state equilibrium with myopic insider displaying full youths unemployment while such a steady state does not exist with FL insiders. Moreover, since equilibria with myopic insiders have unique steady states our multiplicity result is clearly driven by the FL behavior of insiders.

3.3 Steady state equilibria

The next proposition gives the equilibrium prices and quantities for both steady state political-economy equilibria when insiders are forward looking and compares them.

Proposition 4 *For parameters value such that there is multiple steady state, each steady state is associated to a different equilibrium growth path.*

At the full employment steady state political economy equilibrium (sspee) aggregate capital K^*_{fe}, the capital labor ratio k^*_{fe} and aggregate output Y^*_{fe}, are the following:

$K^*_{fe} = C\left(\frac{L^2(x)}{H(1)}\right)^{\frac{1}{1-\alpha}} H(1)$

$k^*_{fe} = C\left(\frac{L^2(x)}{H(1)}\right)^{\frac{1}{1-\alpha}}$

$Y^*_{fe} = C^{\alpha}\left(\frac{L^2(x)}{H(1)}\right)^{\frac{\alpha}{1-\alpha}} H(1)$

At the youths unemployment steady state political economy equilibrium (sspee) aggregate capital K^*_{yu}, the capital labor ratio k^*_{yu} and aggregate output Y^*_{yu}, are the following:

$K^*_{yu} = C\left(\frac{L^2(\frac{1-b}{a})}{H(\frac{1-b}{ax})}\right)^{\frac{1}{1-\alpha}} H(\frac{1-b}{ax})$

$k^*_{yu} = C\left(\frac{L^2(\frac{1-b}{a})}{H(\frac{1-b}{ax})}\right)^{\frac{1}{1-\alpha}}$

$Y^*_{yu} = C^{\alpha}\left(\frac{L^2(\frac{1-b}{a})}{H(\frac{1-b}{ax})}\right)^{\frac{\alpha}{1-\alpha}} H(\frac{1-b}{ax})$

where C is a constant, $C = \frac{\beta}{1+\beta}(1-\alpha)A$,

The following inequalities hold at the steady state:

$$\begin{aligned} k^*_{yu} &> k^*_{fe} \\ K^*_{fe} &> K^*_{yu} \\ Y^*_{fe} &> Y^*_{yu} \end{aligned}$$

Proof. see appendix. ■

According to this proposition while productivity is higher in the low employment equilibrium aggregate output is lower as well as aggregate saving rate. Indeed, the higher productivity is due to higher capital deepening resulting from lower labor utilisation. These results are broadly consistent with empirical studies that compares some European countries to the US. These studies show that til mid 1990's productivity in some European countries (notably France and Germany) was higher than in the US[18]

[18] At least til the mid 1990's, indeed since then hourly productivity in US catch that with europe. The most plausible explanation put forward is that of a lag effect on productivity of investment on ICT made earlier in the US (ref....).

while per capita income remains lower (see Beaudry and Collard 2003; Gordon, 1995). It is interesting to note that while youth unemployment increases insiders income and the short run economy saving rate which may in the short run increase growth in the long run aggregate saving is lower in the economy with unemployment, because of the adverse impact it has on human capital accumulation. This is a similar argument as the one developed by Gordon (1995) whereby the trade-off between productivity and labor is only temporary as higher unemployment decreases workers available income and lowers aggregate savings. Hence the productivity lead of Europe is only temporary since it drives capital depletion in the long run. In my model youth unemployment raises saving in the short run since it redistributes income toward groups of workers with the higher propensity to save. In the long run because its has a negative impact on human capital accumulation unemployment is also detrimental to physical capital accumulation. In the Appendix B I show, within an endogenous growth model based on a physical capital externality *a la* Romer (1989), that in this case steady state growth of productivity is also lower in the high unemployment equilibrium, suggesting that the employment-productivity trade-off may be only temporary.

Some simple equilibrium comparative statics are worth to note. The first refers to the impact of an increase in z and the other to the impact of a change in S. An increase in z can be assimilated to a skill biased technical change, that raises the efficiency of skilled labor compared to the unskilled. We assume that $\Delta\eta = \eta_e - \eta_u$ remains constant, while $\frac{\eta_e}{\eta_u}$ increases. It has been argued that inequality between skilled and unskilled workers have risen in both absolute and relative terms (Caselli, 2000), it will ease the comparative static, without loss of generality to assume that relative inequality $\frac{\eta_e}{\eta_u}$ increases while absolute inequality remains constant. Whether they are in the high or in the low employment equilibrium economies will adapt very differently to such a shock. In the high employment equilibrium following a marginal increase in z, school enrollment raises and, due to higher capital accumulation, productivity rises (k^*_{fe}) also while the economy remains at the full employment equilibrium. Instead, in the low employment equilibrium, for the same rise in the minimum wage future wage schedule is steeper, meaning that the wage gains of future insiders from rising the minimum wage is higher. Hence a moderate increase in z has a negative impact on youth employment rate. The impact on productivity is more ambiguous than previously.

An increase in S may be due to a change in the capital share of income, α. In the high unemployment equilibrium an increase in S rises the equilibrium employment and human capital. Indeed due to a higher marginal productivity of capital, future human capital worth more because the loss of interest rate is higher. In the high employment equilibrium a marginal increase in α has virtually no impact on employment and human capital accumulation.

The general lesson that we want to stress through these straightforward comparative statics is that depending on their initial steady state equilibrium, and human capital stock, economies adapt very differently to the same shocks when one consider endogenous labor market institutions. This may explain why some economies failed to take advantage of the skill biased technological change to rise their stock of human capital. In my model this is due to the uncoordinated behavior of successive generations of insiders. Following the same raise in skill premium some economies adapt by accumulating human capital while in others higher youth unemployment may dampen incentives brought by higher skill premium and human capital may fail to adjust.

In the model the institution through which outsiders labor supply is rationed takes the form of a minimum wage. However I thing that the argument is more general and remains valid for any other policy that increases the relative cost of hiring an outsider compared to an insider as firing and hiring costs. In the model the cost of firing a prime-aged workers has been set to infinity to focused on minimum wage policy. Moreover I have assumed that unemployment creates human capital depletion as it increases the ustility cost of investing in training. One may consider a model where, as in Pissarides (1992), unemployment creates human capital depletion independently of its impact on education per-se. Indeed, it is a well known fact of Mincerian wage regression that experience is positively correlated with productivity (Topel, 1992). Hence if at any age and education levels insiders in the first periods on the labor market experiment unemployment spells, then on average they should be endowed with less human capital than the same insiders who experiment high employment.

4 Conclusion

In most countries unemployment fell disproportionately on youths segment of the labor market. Those have been considered as outsiders in my model. Youth is precisely the life period during which critical human capital is acquired either in school or directly on the labor market. In the context of a three period OLG model I have shown that differences in labor market institutions, labor productivity and per capita income, can be interpreted as politico-economic equilibria of countries with different stock of initial human capital and otherwise identical fundamentals. The optimal wage policy adopted by rational and forward looking insiders face an intergenerational trade-off aimed at maximising their remaining lifetime utility. In countries with high initial human capital, insiders adopt a flat wage profile and follow a human capital driven growth along the transition to a steady state characterised by lower labor productivity and higher income per capita. In countries with lower initial human capital successive generations of insiders adopt a steeper wage profile and the economy has a physical capital driven growth. In this case unemployment experienced by successive generations of young creates human capital depletion, along the transition to a steady state characterised by a higher labor productivity and a lower income per capita.

The model is a step forward to endogenise labor market institutions and investigate their future prospect and impact on growth and productivity. The Overlapping Generation Structure of the model should allow fruitful extensions. In the Appendix B the model is extended to account for endogenous growth. We show that the steady state productivity growth is higher in the high employment equilibrium such that the employment productivity trade-off may be only temporary.

The empirical literature on labor market institutions has mainly been concerned with the impact of those institution on employment and income distribution, by emphasasing its role in the allocation of individuals to jobs. Medium to long run effect of labor market institutions and its impact on youth employment opportunities have received yet little attention (Topel 1999). From an empirical point of view this paper suggests that labor market institutions affecting youth unemployment prospect have important long run effect on growth and productivity. Hence to fully understand the impact of labor market institution from the medium to the long run this paper points out that it is crucial to consider, and control for, their intragenerational short run employment effects on different age groups.

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APPENDIX

A Proofs

Proposition (1)

The first step in the quest of Markov equilibria is to guess the period $t+1$ optimal rule $\tilde{l}_{t+1} = \tilde{L}(E(\tilde{l}_t))$ and verify that this is also the optimal rule followed by current insiders. This optimal rule is characterised in the following lemma:

Lemma 2 *I note* $z \equiv \frac{dE}{dl}(\eta_e - \eta_u) \equiv x\Delta\eta > 0$ *an index measuring the net increase in human capital following a marginal decrease in minimum wages hence this is a measure of the sensitivity of human capital supply to minimum wages, and* $S = \frac{\alpha}{1-\alpha}\left(\frac{1}{\beta} + \alpha\right)$

• *If* $z > \frac{1+\eta_u}{\eta_u - (S-1)}(S-1)$

$$l_{t+1} = 1 \tag{14}$$

• *If* $(S-1) < z \leq \frac{1+\eta_u}{\eta_u-(S-1)}(S-1)$

$$\begin{aligned} \tilde{l}_{t+1} &= \tilde{L}(E(\tilde{l}_t)) = \begin{cases} 1 & if \quad \tilde{l}_t > \frac{1-b}{ax} \\ ax\tilde{l}_t + b & if \quad \tilde{l}_t \leq \frac{1-b}{ax} \end{cases} && (15) \\ \textit{with } a &> 1 \textit{ and } b > 0 && (16) \end{aligned}$$

• *If* $\underline{z} < z \leq (S-1)$ *where* $\underline{z} : (\underline{z})^2 + \underline{z}(1+\eta_s) - \eta_s(S-1) = 0$

$$\begin{aligned} \tilde{l}_{t+1} &= \tilde{L}(E(\tilde{l}_t)) = \begin{cases} ax\tilde{l}_t + b & if \quad \tilde{l}_t > \frac{-b}{ax} \\ 0 & if \quad \tilde{l}_t \leq \frac{-b}{ax} \end{cases} && (17) \\ \textit{with } a &< 1 \textit{ and } b < 0 && (18) \end{aligned}$$

• *If* $z \leq \underline{z}$

$$l_{t+1} = 0$$

and $a = \frac{\Delta\eta}{S-1} > 0$ *and* $b = \frac{\eta_u(z-(S-1))}{(S-1)(z+1)}$

Proof. First step: guess a functional form with unknown parameters for the policy function tomorrow, in a Markov equilibrium this variable depends only on the current state. Here the state variable is the number of generalists E_t. Then, verify that the same function determines present choices.

With restriction to Markov equilibria we just need to anticipate the vote one period ahead. We can guess a linear policy rule for the minimum wage, assume that: $\tilde{L}(E_t) = aE_{t+1} + b$ replacing E_{t+1} by the private rule $E(l_t) = xl_t$ one has:

$$\tilde{l}_{t+1} = \tilde{L}(E_t) = ax\tilde{l}_t + b \tag{19}$$

Hence we can plug in the critical voter choice the optimal policy rule assumed, and the private policy rule which gives the following problem to solve:

$$\max_{\tilde{l}_t} -\alpha(1+\alpha\beta)\ln\left(\tilde{l}_t + \eta_u + E_t\Delta_\eta\right) + \tag{20}$$

$$\beta(1-\alpha)\ln\left(ax\tilde{l}_t + b + \eta_u + E_{t+1}\Delta_\eta\right) \tag{21}$$

$$s.t.\ E_{t+1} = E(l_t) = x\tilde{l}_t \text{ and } \tilde{l}_t \in [0,1]$$

$FOC:$

$\frac{S}{\tilde{l}_t+\eta_u+E_t\Delta_\eta} = \frac{x(a+\Delta_\eta)}{ax\tilde{l}_t+b+\eta_u+x\tilde{l}_t\Delta_\eta}$ with $S = \frac{\alpha(1+\alpha\beta)}{\beta(1-\alpha)} > 1$ (by assumption a sufficient condition is $\alpha > \beta$), this gives:

$$\tilde{l}_t = \frac{\Delta_\eta}{S-1}E_t + \frac{x\eta_u\left(a+\Delta_\eta\right) - S(b+\eta_u)}{x(a+\Delta_\eta)\left(S-1\right)} \tag{22}$$

$a = \frac{\Delta_\eta}{S-1}$ and $b = \frac{x\eta_u(a+\Delta_\eta)-S(b+\eta_u)}{x(a+\Delta_\eta)(S-1)}$

Second step: identify coefficients

$$a = \frac{\Delta_\eta}{S-1}$$

$$b = \frac{\eta_u\left(x\Delta_\eta - (S-1)\right)}{(S-1)\left(x\Delta_\eta+1\right)}$$

Hence since $\Delta_\eta > 0$ and applying the constraint $\tilde{l}_t \in [0,1]$ one has that:

$$\tilde{l}_{t+1} = \tilde{L}(E_t) = \begin{cases} 1 & if \quad \tilde{l}_t > \frac{1-b}{ax} \\ ax\tilde{l}_t + b & if \quad \frac{-b}{ax} \leq \tilde{l}_t \leq \frac{1-b}{ax} \\ 0 & if \quad \tilde{l}_t \leq \frac{-b}{ax} \end{cases}$$

However, not all those choices are mutually consistent for all parameter values, denote $z = x\Delta_\eta > 0$

- $\frac{-b}{ax} < \frac{1-b}{ax} \leq 1 \; iff$

$$k(z) = z^2 - z\left[(S-1) - (1+\eta_s)\right] - (S-1)(1+\eta_s) \geq 0 \tag{23}$$

with a root: $z_1 = (S-1)$

hence $\frac{1-b}{ax} \leq 1$ for all $z \geq (S-1)$.

- $\frac{1-b}{ax} > 0 \; iff$

$$b < 1 \Leftrightarrow z < (S-1)\frac{(1+\eta_u)}{\eta_u - (S-1)} = (S-1)\,c \tag{24}$$

with $c = \frac{1+\eta_u}{1+\eta_u - S} > 1$, hence

$$\frac{1-b}{ax} \in (0,1] \Leftrightarrow (S-1) < z \leq (S-1)\,c \tag{25}$$

- $\frac{-b}{ax} > 0 \; iff \; b < 0 \Longleftrightarrow z < (S-1)$
- $\frac{-b}{ax} \leq 1 \; iff$

$$h(z) = z^2 + z(1+\eta_u) - \eta_u (S-1) \geq 0 \tag{26}$$

with the positive root:

$$\underline{z} = \frac{(1+\eta_u) + \sqrt{(1+\eta_u)^2 + 4\eta_u(S-1)}}{2} \tag{27}$$

then $\frac{-b}{ax} \leq 1$ for all $z > \underline{z}$ Note that $h(S-1) > 0$ hence $(S-1) > \underline{z}$. $\frac{-b}{ax}$ is a valid candidate if the following conditions are met:

$$\frac{-b}{ax} \in (0,1] \Leftrightarrow \underline{z} < z \leq (S-1)$$

- If $z < \underline{z}$ then $\frac{1-b}{ax} > \frac{-b}{ax} > 1$ while if $z > (S-1)\,c$ then $\frac{-b}{ax} < \frac{1-b}{ax} < 0$.

  Henceforth I will focus on cases where interior solutions can merge that is $(S-1) < z \leq (S-1)\,c$ and $\underline{z} < z \leq (S-1)$.

■

For extreme values of z, that is either a high or a low productivity gap between both types of workers, the unique rational expectation regarding next period youths employment rate is a corner solution. If the gain in human capital associated with lower minimum wage is high then the increase in returns to capital outweighed the cost of lower current wage income and insiders choose full employment. This is due to the fact that with a high z human capital lost due to unemployment is high and rate of returns on saving is low. Instead, for a low skill premium (low impact of minimum wage on human capital accumulation) forgone human capital (lower next period interest rate) is more than compensated by the increase in current income. For intermediate ranges of productivity gap, next period youth employment rate depends on current period choice of minimum wage via its impact on next period state variable E_t, this is the essence of Markov equilibria. Once l_{t+1} is determined one can turn to the determination of current period labor market policy $\tilde{l}_t$. We will focus on the case where interior solutions can emerge in steady state equilibrium. Indeed for $z \leq S-1$ the unique steady state equilibrium is full unemployment for the youth, while for $z > (S-1)\,c$ the unique equilibrium which is immediately reached by the first generation of insiders display full employment. For $S-1 \leq z \leq (S-1)\,c$, one has equilibria with a unique or multiple steady states. We prove next the core of proposition (1):

Proof. Assume $(S-1) < z \leq (S-1)\,c$

- Then the optimal policy rule for $t+1$ insiders is:

$$\tilde{l}_{t+1} = \tilde{L}(E(\tilde{l}_t)) = \begin{cases} 1 & if \quad \tilde{l}_t > \frac{1-b}{ax} \\ ax\tilde{l}_t + b & if \quad \tilde{l}_t \leq \frac{1-b}{ax} \end{cases} \tag{28}$$

The insider objective function is not differentiable at $\tilde{l}_t = \frac{1-b}{ax}$. Hence we need to define $\hat{V}^a$ and $\hat{V}^b$ as follows:

$$\begin{aligned} \hat{V}^a &= \max_{\tilde{l}_t \in (\frac{1-b}{ax},1]} -\alpha(1+\alpha\beta)\ln\left(\tilde{l}_t + \eta_u + E_t\Delta_\eta\right) + \beta(1-\alpha)\ln\left(1+\eta_u + E_{t+1}\Delta_\eta\right) \\ st\ E_{t+1} &= xl_t \text{ and } \tilde{l}_{t+1} = \tilde{L}(E_t) = 1 \end{aligned}$$

for an interior solution the optimal policy choice in the set $(\frac{1-b}{ax}, 1]$ is:

$$\hat{l}_t^a = aE_t + b' \tag{29}$$

with $b' = b - \frac{S}{S-1}\frac{(1+\eta_u)-x\Delta_\eta}{x\Delta_\eta(x\Delta_\eta - 1)} < b$ and $b' < 0$ and

$$\hat{V}^a = \begin{cases} \begin{array}{c} -\alpha(1+\alpha\beta)\log\left(1+\eta_u+E_t\Delta_\eta\right)+ \\ \beta(1-\alpha)\log\left(1+\eta_u+x\Delta_\eta\right) \end{array} \equiv \hat{V}^{a,corr2} & if \quad E_t > \frac{1-b'}{a} \\ \begin{array}{c} -\alpha(1+\alpha\beta)\log\left(\hat{l}_t^a+\eta_u+E_t\Delta_\eta\right)+ \\ \beta(1-\alpha)\log\left(1+\eta_u+x\hat{l}_t^a\Delta_\eta\right) \end{array} \equiv \hat{V}^{a,int} & if \quad \frac{1}{a}\left(\frac{1-b}{ax}-b'\right) < E_t < \frac{1-b'}{a} \\ \begin{array}{c} -\alpha(1+\alpha\beta)\log\left(\frac{1-b}{ax}+\eta_u+E_t\Delta_\eta\right)+ \\ \beta(1-\alpha)\log\left(1+\eta_u+\frac{1-b}{a}\Delta_\eta\right) \end{array} \equiv \hat{V}^{a,corr1} & if \quad E_t < \frac{1}{a}\left(\frac{1-b}{ax}-b'\right) \end{cases}$$

and:

$$\hat{V}^b = \max_{\tilde{l}_t \in (0, \frac{1-b}{ax}]} -\alpha(1+\alpha\beta)\ln\left(\tilde{l}_t+\eta_s+G_t\Delta_\eta\right)+\beta(1-\alpha)\ln\left(\tilde{l}_{t+1}+\eta_s+G_{t+1}\Delta_\eta\right)$$

$$st\ G_{t+1} = xl_t \text{ and } \tilde{l}_{t+1} = \tilde{L}(G_t) = ax\tilde{l}_t + b$$

for an interior solution the optimal policy choice in the set $(0, \frac{1-b}{ax}]$ is:

$$\hat{l}_t^b = aE_t + b \tag{30}$$

then,

$$\hat{V}^b = \begin{cases} \begin{array}{c} -\alpha(1+\alpha\beta)\ln\left(\frac{1-b}{ax}+\eta_u+E_t\Delta_\eta\right)+ \\ \beta(1-\alpha)\ln\left(1+\eta_u+\frac{1-b}{a}\Delta_\eta\right) \end{array} \equiv \hat{V}^{b,corr} & if \quad E_t \geq \frac{1}{a}\left(\frac{1-b}{ax}-b\right) \\ \begin{array}{c} -\alpha(1+\alpha\beta)\ln\left(\hat{l}_t^b+\eta_u+E_t\Delta_\eta\right)+ \\ \beta(1-\alpha)\ln\left(l_{t+1}(\hat{l}_t^b)+\eta_u+x\hat{l}_t^b\Delta_\eta\right) \end{array} \equiv \hat{V}^{b,int} & if \quad E_t < \frac{1}{a}\left(\frac{1-b}{ax}-b\right) \end{cases}$$

Note first that since $a > 0$ and $b > 0$, $\hat{l}_t^b > \hat{l}_t^a$, whenever $\hat{l}_t^b$ is an interior solution $\hat{l}_t^a$ is a corner solution and $\hat{V}^{b,corr} = \hat{V}^{a,corr1}$. Hence we have the following result from which we derive the dynamic of minimum wage as well as that of E_t, by applying the law of motion $E_{t+1} = x\tilde{L}(E(\tilde{l}_t))$.

- If $E_t > \frac{1-b'}{a}$ then $\hat{l}_t^a > 1 \Rightarrow \tilde{l}_t^a = 1$ and $\hat{l}_t^b = \frac{1-b}{ax}$ hence $\hat{V}^b = \hat{V}^{b,corr} = \hat{V}^{a,corr1}$ and $\hat{V}^a = \hat{V}^{a,corr2}$, but then one see that $\hat{V}^{a,corr2} > \hat{V}^{b,corr} = \hat{V}^{a,corr1}$ since $\frac{\partial V}{\partial \tilde{l}_t} > 0$ for $\tilde{l}_t < \hat{l}_t^b$, and $\tilde{L}(G_t) = 1$.

- If $\frac{1}{a}\left(\frac{1-b}{ax} - b\right) < \frac{1}{a}\left(\frac{1-b}{ax} - b'\right) < E_t < \frac{1-b'}{a}$ then $\hat{l}_t^b = \frac{1-b}{ax}$ and $\hat{l}_t^a \in (\frac{1-b}{ax}, 1)$. It implies that $\hat{V}^a = \hat{V}^{a,int}$ and $\hat{V}^b = \hat{V}^{b,corr} = \hat{V}^{a,corr1}$ and since $\hat{V}^{a,int} > \hat{V}^{a,corr1}$ the optimal policy is $\tilde{L}(E_t) = \tilde{l}_t^a = \hat{l}_t^a = aE_t + b'$

- If $\frac{1}{a}\left(\frac{1-b}{ax} - b\right) < E_t < \frac{1}{a}\left(\frac{1-b}{ax} - b'\right)$, $\hat{l}_t^b = \frac{1-b}{ax}$ and $\hat{l}_t^a = \frac{1-b}{ax}$ then $\hat{V}^b = \hat{V}^{b,corr} = \hat{V}^{a,corr1}$ the optimal policy is then $L(E_t) = \frac{1-b}{ax}$

- If $E_t < \frac{1}{a}\left(\frac{1-b}{ax} - b\right)$, $\hat{l}_t^b$ is an interior solution and $\hat{l}_t^a = \frac{1-b}{ax}$ then optimal policy is $L(G_t) = \hat{l}_t^b = aE_t + b'$.

  We deduce the following dynamics optimal policy rule:

$$\tilde{L}(E_t) = \begin{cases} 1 & if & & & E_t & > & \frac{1-b'}{a} \\ aE_t + b' & if & \frac{1}{a}\left(\frac{1-b}{ax} - b'\right) & < & E_t & < & \frac{1-b'}{a} \\ \frac{1-b}{ax} & if & \frac{1}{a}\left(\frac{1-b}{ax} - b\right) & < & E_t & < & \frac{1}{a}\left(\frac{1-b}{ax} - b'\right) \\ aE_t + b & if & & & E_t & < & \frac{1}{a}\left(\frac{1-b}{ax} - b\right) \end{cases}$$

  Applying the law of motion $E_{t+1} = E\left(\tilde{L}(E_t)\right) = x\tilde{L}(E_t)$ we deduce the dynamic of human capital accumulation

$$E_{t+1} = \begin{cases} ax\left(E_t - E^*\right) + E^* + cx & \text{if} & & & E_t & < & \frac{1}{a}\left(\frac{1-b}{ax} - b\right) \\ \frac{1-b}{a} & \text{if} & \frac{1}{a}\left(\frac{1-b}{ax} - b\right) & < & E_t & < & \frac{1}{a}\left(\frac{1-b}{ax} - b'\right) \\ ax\left(E_t - E^*\right) + E^* & \text{if} & \frac{1}{a}\left(\frac{1-b}{ax} - b'\right) & < & E_t & < & \frac{1-b'}{a} \\ x & \text{if} & & & E_t & > & \frac{1-b'}{a} \end{cases} \tag{31}$$

■

We can have two economies with exactly the same stock of aggregate quantity of human capital, $\eta_u + E_t\Delta\eta$, the one the sensitivity of human capital supply to the minimum wage is more likely to converge toward a full employment equilibrium. Consider two economies that are identical but have slightly different initial level of general human capital in the neighborhood of E^*. The economy with $E < E^*$ converges (cv) toward a steady state equilibrium with employment, while the economy with $E > E^*$ cv toward an economy with full employment.

Proof. Set of equilibria in proposition 4

- According to the dynamics in (31), a sufficient condition for multiple equilibria is:

$$\frac{1}{a}\left(\frac{1-b}{ax} - b'\right) < E^* = \frac{b'x}{1-ax} < x \text{ and } b' < 0$$

where E^* is the fix point of the recurrent equation $E_{t+1} = axE_t + b'x$. Depending on parameters value three possible equilibrium regimes can emerge, two entails a unique steady state and one has multiple steady states:

$$E^* < \frac{1}{a}\left(\frac{1-b}{ax} - b'\right) \text{ then we have a unique equilibrium full employment}$$

$$\frac{1}{a}\left(\frac{1-b}{ax} - b'\right) \quad < \quad E^* = \frac{b'x}{1-ax} < x \text{ then we have multiple equilibrium one with full employment and the other with unemployment}$$

$$E^* = \frac{b'x}{1-ax} > x \text{ then we have a unique equilibrium with unemployment}$$

We remind that $E^* = \frac{b'x}{1-ax}$ is the point corresponding to the intersection of the third segment $E_t = ax + b'$ (see fig.*1, 2, 3* in the text*)* and the 45° line.

The right hand side condition for multiple equilibria is:

$$\frac{b'x}{1-ax} < x \Leftrightarrow b' + ax > 1$$

Replacing b' and ax by their respective values and with $z > S - 1$ the condition is:

$$\begin{aligned} b' + ax &> 1 \\ &\Leftrightarrow H(z) = (z - S)(Z + \eta_u + 1) > 0 \\ &\Leftrightarrow z > S \end{aligned} \tag{32}$$

such a z exists iff $S < c(S-1) \Leftrightarrow \eta_u < (S-1)(S+1)$

The condition on the left side of G^* is $G^* > \frac{1}{a}\left(\frac{1-b}{ax} - b'\right)$

$$\begin{aligned} E^* &> \frac{1}{a}\left(\frac{1-b}{ax} - b'\right) = \underline{E} \Leftrightarrow \\ K(z) &= (S-1)z^2 - z\left(1 + (S-1)\left((S-1) - \eta_u\right)\right) - (1+\eta_u)\left(S + (S-1)^2\right) < 0 \end{aligned}$$

if $\eta_u > (S-1)S$, $K(z) < 0$ then $\forall z \in [(S-1), c(S-1)]$, $E^* > \underline{E}$

if $\eta_u \leq (S-1)S$, $E^* > \underline{E} \Leftrightarrow z < z_h \in ((S-1), c(S-1))$ and $S < z_h$ since $K(S) < 0$ and $K(c(S-1)) > 0$.

Depending on the value of η_u and z the following equilibria are possible

- If $(S-1) < \eta_u < (S-1)S < (S-1)(S+1)$

  - if $S-1 < z < S < z_h < c(S-1)$ then $E^* > x > \underline{E}$ and the economy converges to an **equilibrium with unemployment** and $E^* = \frac{1-b}{a}$.

  - if $S < z < z_h < c(S-1)$ then $\underline{E} < E^* < x$ and the economy has **multiple equilibria**, one unstable equilibrium and two stable equilibria, one with full employment and the other with positive youth unemployment rate.

  - if $S < z_h < z < c(S-1)$ then $E^* < \underline{E} < x$ and the the economy converges to a unique full **employment equilibrium.**

- If $(S-1)S < \eta_u < (S-1)(S+1)$

  - if $S-1 < z < S < c(S-1)$ then $E^* > x > \underline{E}$ and the the economy converges to an **equilibrium with unemployment** and $E^* = \frac{1-b}{a}$.

  - if $S < z < c(S-1)$ then $\underline{E} < E^* < x$ has **multiple equilibria**, one unstable equilibrium and two stable equilibria, one with full employment and the other with positive youths unemployment rate.

- If $(S-1)S < (S-1)(S+1) < \eta_u$ the economy converges to an **equilibrium with unemployment** and $E^* = \frac{1-b}{a}$.

■

The following proposition sum-up, which is stated in the text as proposition (2) sumarises the results

Proposition

$\forall S > 1$ there is a $\underline{\eta_e} = (S-1)(S+1)$ such that if $\eta_e < \underline{\eta_e}$ then there exist a z_h and a z_l with $z_h > z_l$ and $z_h, z_l \in ((S-1), c(S-1))$ such that:

- *if $z < z_l$ the economy converges to a unique equilibrium with unemployment*
- *if $z > z_h$ the economy converges to a unique equilibrium with full employment*
- *if $z_l < z < z_h$ the economy has multiple equilibria, an unstable equilibrium and two stable equilibria, one with unemployment and the other with full employment.*

Equilibria with myopic insiders:

Proof. Proposition (3)

Myopic insiders take the next period expected employment rate as a parameter $\bar{l}^a_{t+1}$ and choose $\tilde{l}^{my}_t$ such that :

$$\begin{aligned} \tilde{l}^{my}_t &= \arg\max_{\tilde{l}_t \in [0,1]} V^{my}(\tilde{l}_t; E_t; \tilde{l}^a_{t+1}) = -S \ln\left(\tilde{l}_t + \eta_e + E_t \Delta_\eta\right) + \ln\left(\bar{l}^a_{t+1} + \eta_e + E_{t+1}\Delta_\eta\right) \\ st\ E_{t+1} &= x l_t \end{aligned}$$

The FOC is:

$$\frac{S}{\tilde{l}_t + \eta_u + E_t \Delta_\eta} = \frac{z}{\bar{l}^a_{t+1} + \eta_u + E_{t+1}\Delta_\eta} \tag{33}$$

$$\tilde{l}^{my}_t(E_t, \tilde{l}^a_{t+1}) = \frac{\Delta\eta}{S-1} E_t + \frac{\eta_u (z - S) - S\bar{l}^a_{t+1}}{(S-1)\,z}$$

One can check that

$$aE_t + b' < \tilde{l}^{my}_t < aE_t + b$$

- We shall establish the following:

  (i) if agents are myopic there is no interior solution with steady state solution, hence steady state solutions with myopic behavior are corner solutions

  (ii) corner solution involve monotonic sequence of employment rates

  (iii) If $z \geq S$ the sequence of employment rate in increasing and bounded from above by 1

  (iii) If $z < S$ the sequence of employment rate decreasing and bounded from below by 0

- To check that there is no interior solution simply not that a fix point of (33) is the solution to:

$$\tilde{l}^{my}(E(x), x) = x$$

  which is $x = -\frac{\eta_u}{z+1} < 0$, hence a steady state is necessarily a corner solution.

  (i) Assume that $\tilde{l}^a_{t+1} = 1$ then we should show that $\frac{\partial V(\tilde{l}_t; x; 1)}{\delta \tilde{l}_t} > 0 \forall \tilde{l}_t \in [0, 1]$. Namely, when the current state of the economy is such that the proportion of educated insiders is compatible with full employment, then insiders expecting full employment will also choose full employment. Indeed one can check that:

$$\tilde{l}^{my}_t(x, 1) = \frac{\Delta \eta}{S-1} x + \frac{\eta_u (z-S) - S}{(S-1) z} \geq 1 \Leftrightarrow (z-S)(z+1+\eta_u) \geq 0$$

  hence $\tilde{l}^{my}_t = 1$ is a rational expectation equilibrium if $z \geq S$

  (ii) Assume that $\tilde{l}^a_{t+1} = 0$ then $\tilde{l}^{my}_t = 0$ is a steady state equilibrium if $\tilde{l}^{my}_t(0, 0) \leq 0$ which is true iff $z < S$. Rather if $z \geq S$ then the policy $\tilde{l}^{my}_t = 0$ is never a steady state rational expectation with myopic insiders.

- Rational expectations are monotonic. This follows from the observation made in the text in fig 4 and fig. 5. Namely if a current insider find it profitable to increase minimum wages then it is still more profitable for the next period insiders to keep increasing minimum wages. While if it were profitable for insiders to lower minimum wages it still more profitable for the next period insiders to keep doing so. If expectations are rational then they should verify that if $l_t > l^a_{t+1}$ then $l_{t+1} > l^a_{t+2} = l_{t+2} > ..l_{t+j} = l_{t+j+1}$ and if $l_t < l^a_{t+1}$ then $l_{t+1} < l^a_{t+2} = l_{t+2} < ..l_{t+j}$.

- Assume $z < S$ and that $l^a_{t+1} < \tilde{l}^{my}_t$ then with rational expectations $E_{t+2} < E_{t+1}$ and by backward induction $E_{t+1} < E_t$, which is compatible with (33), hence by induction we can find a sequence of interior solution starting from $\tilde{l}_t$ such that: $l_t > l^a_{t+1} = l_{t+1} > l^a_{t+2} = l_{t+2} > ..l_{t+j} = l_{t+j+1}$ are all rational expectations equilibrium, this sequence is decreasing and bounded hence it converges to $\tilde{l}^{my}_t = 0$ which have been shown is a steady state rational expectation equilibrium if $z < S$.

  Assume $z < S$ and that $l^a_{t+1} > \tilde{l}^{my}_t$ then with rational expectation $E_{t+2} > E_{t+1}$ and by backward induction $E_{t+1} > E_t$, inspecting (33) one has that $\frac{S}{\tilde{l}_t+\eta_e+E_t\Delta_\eta} > \frac{z}{\tilde{l}^a_{t+1}+\eta_e+E_{t+1}\Delta_\eta}$ in this case and for all $\tilde{l}^{my}_t$ such that $\tilde{l}^{my}_t < l^a_{t+1}$ is true, the optimal choice is $\tilde{l}^{my}_t = 0$ (since the objective function is decreasing in $[0, \tilde{l}^a_{t+1}]$).

  Hence if $z < S$ the economy with myopic has a unique steady state equilibrium with $\tilde{l}_t = 0$.

- Assume now that $z > S$ and that $l^a_{t+1} \leq \tilde{l}_t$ then with rational expectation $E_{t+2} < E_{t+1}$ and by backward induction $E_{t+1} < E_t$, inspecting (33) one has that $\frac{S}{\tilde{l}_t+\eta_e+E_t\Delta_\eta} < \frac{z}{\tilde{l}^a_{t+1}+\eta_e+E_{t+1}\Delta_\eta}$ in this case and for the set of policies such that $\tilde{l}^a_{t+1} < \tilde{l}^{my}_t$ the optimal choice is $\tilde{l}^{my}_t = 1$, since the objective function is increasing in $[\tilde{l}^a_{t+1}, 1]$ for $z > S$.

  \- Assume rather that $l^a_{t+1} > \tilde{l}_t$ then with rational expectations $E_{t+2} > E_{t+1}$ and by backward induction $E_{t+1} > E_t \Leftrightarrow \tilde{l}_t > \tilde{l}_{t-1}$ clearly there exists an interior $\tilde{l}_t$ compatible with the FOC (33) by induction one can construct an increasing rational expectation sequence $l_t < l^a_{t+1} = l_{t+1} < l^a_{t+2} = l_{t+2} < l^a_{t+3} = l_{t+3} < ... < l^a_{t+j} = 1 = l_{t+j} = l_{t+j+1}$, which is bounded by $\tilde{l}_t = 1$ hence starting from

$l^a_{t+1} > \tilde{l}_t$ the economy converges to $\tilde{l}_t = 1$. And we have shown that $\tilde{l}^{my}_t = 1$ is a steady state rational expectation equilibrium.

To sum-up

- With $z > S$

  The unique steady state rational expectation equilibrium with myopic insider display full employment.

- With $z < S$,

  The unique steady state rational expectation equilibrium with myopic insider display full unemployment for youths.

■

Proof. Proposition (4), Assume $(S-1) < z \leq (S-1)\,c$ and that $\eta_s < (S-1)\,(S+1)$ The capital stock evolves according to the following law of motion:

$$K_{t+1}(E_t) = \frac{\beta}{1+\beta} A(1-\alpha) \left(\frac{K_t}{H(\tilde{L}(E_t); E_t)} \right)^{\alpha} * L^2_t(E_t)$$

We now that the policy variable converges to two possible steady state equilibrium, one with full employment, $\tilde{L}(E_t) = \tilde{L}(E_{t+1}) = 1$ and $E_t = E_{t+1} = x$, and the other with youths being partly unemployed, $\tilde{L}(E_t) = \tilde{L}(E_{t+1}) = \frac{1-b}{ax}$ and $E_t = E_{t+1} = \frac{1-b}{a}$. Each steady state policy corresponds to different steady state equilibrium growth path for the capital stock:

$$K^{fe}_{t+1} = \frac{\beta}{1+\beta} A(1-\alpha) \left(\frac{K^{fe}_t}{H(1)} \right)^{\alpha} * L^2_t(x)$$

$$K^{yu}_{t+1} = \frac{\beta}{1+\beta} A(1-\alpha) \left(\frac{K^{yu}_t}{H(\frac{1-b}{ax})} \right)^{\alpha} * L^2_t(\frac{1-b}{a})$$

From these laws of motion it is straightforward to deduce the corresponding steady state aggregate capital, capital labor ratio, and aggregate output.

- $\left(K^*_{fe}\right)^{1-\alpha} = \left(\frac{1}{1+\eta_e+\Delta\eta x}\right)^{\alpha}(\eta_e+\Delta\eta x) > \left(\frac{1}{\frac{1-b}{ax}+\eta_e+\Delta\eta\frac{1-b}{a}}\right)^{\alpha}\left(\eta_e+\Delta\eta\frac{1-b}{a}\right) = \left(K^*_{yu}\right)^{1-\alpha}$ to compare these expressions let's define the following useful function:

  $$\ln(f(u)) = -\alpha\ln(\frac{u}{x}+\eta_e+\Delta\eta u)+\ln\left(\eta_e+\Delta\eta u\right)$$

  and $u \in D = [\frac{1-b}{a}, x]$, one see that $f(\frac{1-b}{a}) = \left(K^*_{yu}\right)^{1-\alpha}$ and $f(x) = \left(K^*_{fe}\right)^{1-\alpha}$ to prove that $K^*_{fe} > K^*_{yu}$ we shall show that the function is increasing over the domain D, that is

  $$\alpha\frac{1/x+\Delta\eta}{\frac{u}{x}+\eta_e+\Delta\eta u} < \frac{\Delta\eta}{\eta_e+\Delta\eta u}$$

  a sufficient condition for the previous inequality to hold is that $\alpha < (1-\alpha)\,x\Delta\eta \equiv (1-\alpha)\,z$ since we assumed that $z \in ((S-1), c(S-1))$ it suffices to show that is true for $S-1$, $\frac{\alpha}{1-\alpha} < S-1$ replacing S by its value this is equivalent to $1 < \frac{\alpha*(1+\alpha\beta)}{\beta}$ a sufficient condition[19] is that $\alpha > \beta$ which is indeed a sufficient condition for $S > 1$.

- We show next that steady state labor productivity that is the capital labor ratio (productivity per hour worked) is higher in the unemployment equilibrium:

  $$k^*_{yu} = \frac{K^*_{yu}}{H(\frac{1-b}{ax})} > k^*_{fe} = \frac{K^*_{fe}}{H(1)} \Leftrightarrow \frac{L^2(\frac{1-b}{a})}{H(\frac{1-b}{ax})} > \frac{L^2(x)}{H(1)}$$

  $\Leftrightarrow$

  $$\frac{1+\eta_e+\Delta\eta x}{\eta_e+\Delta\eta x} > \frac{\frac{1-b}{ax}+\eta_e+\Delta\eta\frac{1-b}{a}}{\eta_e+\Delta\eta\frac{1-b}{a}}$$

  We use the same trick as before and define the function $g(u) = \frac{\frac{u}{x}+\eta_e+\Delta\eta u}{\eta_e+\Delta\eta u}$ on the domain $D = [\frac{1-b}{a}, x]$ and by noting that $g(x)$ is equal to the LHS of the inequality, we have to show that $g' > 0$ for the inequality to hold, indeed we can check that $g'(u) = \frac{\eta_e/x}{(\eta_e+\Delta\eta u)^2} > 0$. This proves that $k^*_{yu} > k^*_{fe}$.

[19] Indeed this is also a sufficient condition for the economy to be dynamically efficient if capital market were perfect.

- Proving $Y^*_{fe} > Y^*_{yu}$ is straightforward owing to the Cobb Douglas production function,

$$Y^*_{fe} > Y^*_{yu} \Leftrightarrow L_{fe} * \left(\frac{K^*_{fe}}{L_{fe}}\right)^{\alpha} > L_{yu} * \left(\frac{K^*_{yu}}{L_{yu}}\right)^{\alpha}$$

with more capital and more labor clearly $Y^*_{fe} > Y^*_{yu}$.

■

Comparative static with respect to z and S

- High unemployment equilibrium

  Change in z

In the high unemployment equilibrium the stock of educated insiders labor is $\frac{1-b}{a}$ and the equilibrium youth employment rate is:

$$\frac{1-b}{ax} = \frac{(S-1)(z+1) - \eta_u (z - (S-1))}{(z+1)z} \tag{34}$$

taking the derivative with respect to z :

$$\frac{-z^2((S-1)-\eta_u) - (2z+1)(S-1)(1+\eta_u)}{(z+1)z} < 0$$

Hence a marginal increase in z increases equilibrium minimum wage.

Change in S

Observing (34) it is clear that as S increases the equilibrium preferred youth employment rate rises, consequently the equilibrium human capital human capital increases.

- High employment equilibrium

  Change in z

In the high employment the equilibrium policy is simply $\tilde{l} = 1$ and the enrollment rate is x, which is an increasing function of z.

Change in S

Observing (34) it is clear that as S increases the equilibrium preferred youth employment rate rises, consequently the equilibrium human capital human capital increases.

Effect on productivity

$k = \left(\frac{\eta_e + \Delta\eta x}{1+\eta_e+\Delta\eta x}\right)^{\frac{1}{1-\alpha}}$ clearly as z increases x increases and the productivity increases.

B Minimum wage policy in an endogenous growth model

Suppose that A rather than being a parameter is endogenous and given by:

$$A_t = AK_t^{1-\alpha} \tag{35}$$

Thus the state of the technology rather than being fix evolves as a function of the aggregate level of capital. Equation (35) can be motivated by the assumption of production externalities. Individuals firms behave competitively and maximise profits taking A_t as given. This leads to an aggregate production function featuring constant marginal return to capital:

$$Y = AK_tH_t^{1-\alpha}$$

As in Romer (1989) production is linear in capital, it is indeed and AK type model of endogenous growth. In this case the equilibrium factor prices are:

$$W_t = (1-\alpha)\,A\frac{K_t}{H_t^{\alpha}}$$

and

$$R_t = \alpha AH_t^{1-\alpha}$$

The capital market equilibrium condition implies:

$$L_t^i * \frac{\beta}{1+\beta}(1-\alpha)\,A\frac{K_t}{H_t^{\alpha}} = K_{t+1}$$

Hence a growth factor during the transition to the steady state growth is:

$$\frac{Y_{t+1}}{Y_t} = L_t^i\frac{\beta}{1+\beta}\frac{(1-\alpha)A}{H_t}H_{t+1}^{1-\alpha} \tag{36}$$

Hence the economy with the higher employment rate has also the higher growth rate.

The insider's objective function is:

$$\begin{aligned} V^2(\tilde{l}_t;E_t) &= -\hat{S}\ln H_t(\tilde{l}_t,E_t) + \ln H_{t+1}(\tilde{l}_{t+1}(\tilde{l}_t),E(\tilde{l}_t)) \\ \text{with } \tilde{l}_t,\ \tilde{l}_{t+1} &\in [0,1] \text{ and } E_{t+1} = x\tilde{l}_t \end{aligned}$$

The problem is the same as the one solved in the main text with $\hat{S} = \frac{\alpha(1+\beta)}{\beta(1-\alpha)} > S$, replacing S.

Since $\hat{S} = \frac{\alpha(1+\beta)}{\beta(1-\alpha)} > S$, a necessary condition for high employment equilibrium is $S < \hat{S} < z$, hence the case for high unemployment equilibrium is more likely with capital externality in production. We can also show that steady state growth rate is higher in the high employment equilibrium:

$$Y_{t+1}/Y_t = L_t^i \frac{\beta}{1+\beta} \frac{(1-\alpha)\, A H_{t+1}^{1-\alpha}}{H_t}$$

at the high employment steady state $H_{t+1} = H_t = H(1)$ and $L_t^i = L^i(x)$, while in the low employment steady state $H_{t+1} = H_t = H(\frac{1-b}{ax})$ and $L_t^i = L^i(\frac{1-b}{a})$. The steady state growth rate in the high employment equilibrium is:

$$(Y_{t+1}/Y_t)^e = L^i(x) \frac{\beta}{1+\beta} \frac{(1-\alpha)\, A}{(H(1))^\alpha}$$

while in the low employment equilibrium it is:

$$(Y_{t+1}/Y_t)^u = L^i(\frac{1-b}{a}) \frac{\beta}{1+\beta} \frac{(1-\alpha)\, A}{\left(H(\frac{1-b}{ax})\right)^\alpha}$$

$$(Y_{t+1}/Y_t)^e > (Y_{t+1}/Y_t)^u \Leftrightarrow \frac{\eta_u + \Delta\eta x}{(1+\eta_u+\Delta\eta x)^\alpha} > \frac{\eta_u + \Delta\eta \frac{1-b}{a}}{\left(\frac{1-b}{ax} + \eta_u + \Delta\eta \frac{1-b}{a}\right)^\alpha}$$

which is the case provided that $z > \frac{\alpha}{1-\alpha}$ The parameter range for multiple steady state implies that $z > \hat{S}$ and since $\hat{S} > \frac{\alpha}{1-\alpha}$ one has $z > \frac{\alpha}{1-\alpha}$. We conclude that the steady state growth rate is higher in the high employment equilibrium.