Household consumption choices in the presence of several decision makers *

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Abstract

A new theoretical framework has recently emerged for analyzing the behavior of households with two spouses. This approach, known as the collective model, assumes that each spouse has individual preferences and that households' outcomes are Pareto efficient. However, the empirical work on collective models that has been performed to date has principally focused on households with two decision makers and ignored the behavior of those with a potentially larger number (*e.g.*, couples living with adult children or with the elderly in developed countries, extended families in developing countries). The goal of this article is twofold: First, we summarize the main tests having been proposed to empirically verify the restrictions of the collective model in this context. We also suggest a test that proves equivalent to another test found in the literature, but that can be easier to implement in some instances. Second, we test a collective model with multiple decision makers using micro-data from a British Survey. The sample used in the estimations includes couples with one child over 16 years old. Our results reject the collective model for one or two decision makers, but not for three decision makers.

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1 Introduction

Modern microeconomic theory generally treats the household as the basic decision-making unit. Decisions of the household, even when it consists of several adults, are assumed to result from the maximization of a standard family utility function subject to a budget constraint. This characterization of the household, commonly called the "unitary model", has provided the basis for nearly all theoretical and empirical work in recent years.

The unitary model owes its popularity to the fact that it allows all the analytical tools developed for the model of the consumer to be used. Thus, it can easily be adapted to a broad range of situations (labor supply, demand, etc.). This flexibility also carries over to empirical analysis. Nonetheless, there is a price to pay for this degree of versatility. In particular, it takes no account of the individual preferences of the agents comprising the household, or of their interactions in the decision-making process. Consequently, the unitary model cannot be used to analyze the distribution of welfare among the members of a household. This is doubtlessly its greatest flaw.

Several authors have attempted to reconcile the existence of individual preferences with the characterization of the household presented by the unitary model. Thus, Samuelson (1956) assumes that household members first reach an agreement regarding the distribution of resources amongst themselves. This consensus then allows their preferences to be aggregated and justifies representing the household with a single utility function, which corresponds to a fixed weighted sum of the individual utilities. However, his model tells us nothing about the creation and stability of this consensus. Becker (1974) was able to present a more convincing characterization of the unitary model with his *Rotten-Kid Theorem*. In his model, the household consists of a head who is altruistic and one or more egoistic individuals. The altruist ensures that the egoists maximize their utility functions by transferring wealth to them. However, this model is very restrictive since it assumes that these transfers are quite extensive (there are no corner solutions) and also that utilities are transferable between the members of the household.

These attempts at reconciling methodological individualism and the unitary model are unsatisfactory, in that they assume a fixed weighting of individual utilities within the family utility function (Samuelson, 1956) or else demonstrate the existence of such a function within a very restrictive context (Becker, 1974). Dissatisfaction vis-à-vis the unitary model has also emerged in the empirical literature. Indeed, several fundamental predictions of the unitary model have not been corroborated empirically. In particular, income pooling[1] and the properties of the Slutsky matrix (symmetry and negativity) are nearly always rejected (e.g. Schultz, 1990; Thomas, 1990; Thomas, 1993; Fortin and Lacroix, 1997; Browning and Chiappori, 1998; Phipps and Burton, 1998).

To respond to the methodological and empirical problems associated with the unitary model, Chiappori and his collaborators have developed a new approach based on the assumptions that decision makers within the household have their own preferences and that the familial decision process leads to outcomes that are weakly Pareto efficient.

[1]Income pooling implies that only the household's total income, and not its distribution between the individual members, should affect decisions. Rejecting this assumption has significant consequences for the conduct of economic and social policy.

These assumptions, which seem relatively innocuous in the framework of the household,[2] actually impose restrictions on behavior that are falsifiable. This characterization of the household, commonly referred to as the "collective model," was initially applied to a labor supply model in an environment that is static and characterized by egoism or "Beckerian caring preferences" (Chiappori, 1988, 1992). Retaining the static context, Bourguignon *et al.* (1995) extended the model to the household's demand for consumption goods. Furthermore, Browning and Chiappori (1998) developed a more general model of household consumption in which relative prices are variable and which accounts for externalities and public goods at the family level. In particular, they generalize the Slutsky conditions and present empirical results for Canadian (FAMEX) data that corroborate the restrictions derived from the general collective model.

In two recent papers, Chiappori and Ekeland (2003,2005) made further breakthroughs by generalizing the work of Chiappori and Browning (1998) in several directions. In their 2005 article, Chiappori and Ekeland present the general characteristics of, and the restrictions that must be placed on, the aggregate demand functions of a group with a fixed and exogenous number of members. Furthermore, in their complementary 2003 paper, they show that, under certain conditions, observations on this type of group allow all, or some, of the individual preferences of the members to be recovered, along with the decision-making process.

This new theoretical framework for analyzing the behavior of households (and groups in general) provides a serious alternative to the unitary analytical framework.[3] However, on the empirical front, aside from a handful of recent studies dealing with polygamous families (Dauphin, Fortin and Lacroix, 2003; Dauphin, 2003) or extended-family households (Rangel 2004), work on collective models has so far been limited to households with two decision makers. Consequently, it has completely ignored the behavior of households in which there may be more. This is a serious limitation, considering the relatively high number of couples living with adult children or the elderly in developed countries and the prevalence of extended families (including polygamous and polyandrous families) in the developing world.

The goal of this paper is twofold. First, we summarize the main tests that have been proposed to empirically verify the restrictions of the collective model in a context in which the household may contain more than two decision makers. We also propose a test based upon distribution factors that proves equivalent to another test featured in the literature, but that can be easier to implement in some instances.[4] Second, we conduct a number of tests on a sample drawn from a series of cross-sectional data (1982–1993) from the British Family Expenditure Survey (FES). The retained sample is limited to couples with one child over 16 years old. These data contain information on prices and some distribution factors. Thus, they allow the theoretical contributions of Browning

[2] If individuals can increase the welfare of their spouses without diminishing their own welfare, why would they not do so? To ask this question is practically to answer it (except, of course, in the event of spouses who do not have a minimum of altruism towards each other or in the presence of incomplete information about each other preferences.)

[3] See Chiappori and Donni (2005) for an excellent overview of collective models and, more generally, non-unitary models

[4] Distribution factors are variables, such as a spouse' relative contribution to the household's income, that affect the decision making process (and thus the intrahousehold choices) without influencing individual preferences or the household's aggregate budget constraint.

and Chiappori (1998) and Chiappori and Ekeland (2003, 2005) to be used for validating the collective model. Our results reject collective rationality with two decision makers, but not with three. Our analysis not only illustrates the importance of accounting for individual preferences in the analysis of households, but also of recognizing that several members participate in the intrahousehold decision making process.

2 The Theoretical Framework

Consider a household comprising $S+1$ ($S \geq 1$) members who participate in the household's consumption decision process,[5] where S is exogenous. Each decision maker i, $i = 1, \ldots, S+1$, has his or her own preferences defined over N market goods consumed by the household. This is represented by a utility function $U_i(\boldsymbol{x})$, that is concave and twice continuously differentiable, where $\boldsymbol{x} \equiv [x_1, \ldots, x_N]$. We impose no restrictions on the nature of these goods: They may be private, public, or characterized by externalities. The household faces a price vector $\boldsymbol{\pi} \in \mathbb{R}^N$. Thus, its budget constraint is:

$$\boldsymbol{\pi}'\boldsymbol{x} = m, \tag{1}$$

where m represents the household's income, assumed exogenous. To simplify notation, we normalize $m = 1$ hereafter. The collective model assumes that consumption choices are efficient. More formally, we postulate the following axiom (known as "collective rationality"):

Axiom 1 *The decision-making process that determines the household's consumption basket leads to weakly Pareto-efficient choices. In other words, for any price vector* $\boldsymbol{\pi}$ *and income* m *(with* $m = 1$*), the consumption vector* $\boldsymbol{x}$ *chosen by the household is such that no other vector* $\overline{\boldsymbol{x}}$ *that satisfies the condition* $\boldsymbol{\pi}'\overline{\boldsymbol{x}} = 1$ *can increase the welfare of all members.*

Thus, the household's decisions do not generally depend only on preferences, income, and prices. They also depend on each member's decision-making power. Consequently, all the factors —the "Extra-household Environmental Parameters" (EEPs) in the terminology of McElroy (1990)—that may contribute to the negotiating power of the household members can affect the outcome of the negotiation process.

Axiom 2 *The decision process depends on* K *variables* $\boldsymbol{y} \equiv [y_1, y_2, \ldots, y_K]'$ *(distribution factors) that are independent of individual preferences and do not globally modify the household's budget constraint.*

There are several examples of distribution factors in the literature: divorce-related legislation, the relative proportion of men and women on the marriage market (Chiappori, Fortin and Lacroix, 2002), and the relative income shares of the household's members.

[5] $S+1$ does not necessarily represent the size of the household. It is possible that there be other household members who do not participate in the decision making.

Assuming that Axioms 1 and 2 are satisfied, we finally postulate the existence of S continuously differentiable scalar functions $\mu_1(\boldsymbol{y},\boldsymbol{\pi}) \geq 0, \ldots, \mu_S(\boldsymbol{y},\boldsymbol{\pi}) \geq 0$, such that the optimization program the household must solve can be written:

$$\underset{\boldsymbol{x}\in\mathbb{R}_+^N}{Max} \sum_{i=1}^{S} \mu_i(\boldsymbol{y},\boldsymbol{\pi}) U_i(\boldsymbol{x}) + U_{S+1}(\boldsymbol{x}), \tag{P}$$

subject to

$$\boldsymbol{\pi}'\boldsymbol{x} = 1,$$

where $\mu_i(\boldsymbol{y},\boldsymbol{\pi})$, $i = 1, \ldots, S$, represents the (Pareto) weights associated with the utility function of decision maker i relative to that of decision maker $S+1$. This is interpreted as an indicator of the power of negotiation or persuasion of the i-th household member.

The solution to program (P) may be obtained in two steps. First, the budget constraint and utility functions determine the household's Pareto frontier. Axiom 1 implies that the outcome of the decision process is located on this Pareto frontier. Second, the vector $\boldsymbol{\mu}(\boldsymbol{y},\boldsymbol{\pi}) = [\mu_1(\boldsymbol{y},\boldsymbol{\pi}), \ldots, \mu_S(\boldsymbol{y},\boldsymbol{\pi})]$ of Pareto weights determines the point chosen on this frontier.

It is important to recognize that the vector $\boldsymbol{\mu}(\boldsymbol{y},\boldsymbol{\pi})$ is generally not constant since it depends on distribution factors, prices, and income (normalized to 1). Consequently, the distribution factors only affect household demand through their impact on the Pareto weights $\boldsymbol{\mu}(\boldsymbol{y},\boldsymbol{\pi})$. Thus, distribution factors do not alter the Pareto frontier.

We denote the vector of Marshallian demands obtained by solving (P) for given values of the weights $\boldsymbol{\mu}$ as $\tilde{\boldsymbol{\xi}}(\boldsymbol{\pi},\boldsymbol{\mu})$. Replacing these weights with their function $\boldsymbol{\mu}(\boldsymbol{y},\boldsymbol{\pi})$, the demand system can be written: $\tilde{\boldsymbol{\xi}}(\boldsymbol{\pi},\boldsymbol{\mu}(\boldsymbol{\pi},\boldsymbol{y}))$. Unfortunately, these demands are unobservable, since the Pareto weights are unobservable. Only their reduced form $\boldsymbol{\xi}(\boldsymbol{\pi},\boldsymbol{y})$ are observable. A fundamental question raised by the collective model is: Given the following equality, does the assumption of collective rationality impose falsifiable empirical restrictions on the observed behavior of the household:

$$\boldsymbol{\xi}(\boldsymbol{\pi},\boldsymbol{y}) = \tilde{\boldsymbol{\xi}}(\boldsymbol{\pi},\boldsymbol{\mu}(\boldsymbol{\pi},\boldsymbol{y})) \ ? \tag{2}$$

A key contribution of Chiappori (1988, 1992) was to demonstrate that, under certain additional conditions,[6] the assumption of collective rationality effectively generates testable restrictions on the behavior of households.[7] The recent literature has generalized this result to the case in which no restrictions are placed on the nature of the goods or the number of decision makers, and has proposed several tests for the collective model when either prices or distribution factors are variable.

[6]He assumes two decision makers, two assignable goods (each decision maker's leisure), a non-assignable good (household consumption), preferences that are egoistic or represent "Beckerian caring preferences," the absence of public goods, and variable prices (wages).

[7]Chiappori also demonstrates that, under these assumptions, it is possible to recover the sharing rule for the couple's exogenous income up to a constant, as well as the preferences of each decision maker, conditional on this constant. In this paper we limit our analysis to the restrictions imposed by the general version of collective model. In this case, it is usually impossible to identify individual preferences or the decision-making process.

2.1 Tests on Prices

A first restriction of the collective model pertains to the price effects of households' aggregate demands.

Proposition 1 *(the SR(S) condition): If $\boldsymbol{\xi}(\boldsymbol{\pi}, \boldsymbol{y})$ solves the program (P), then the Slutsky matrix associated with $\boldsymbol{\xi}(\boldsymbol{\pi}, \boldsymbol{y})$, or $\boldsymbol{S}(\boldsymbol{\pi}, \boldsymbol{y}) = (D_{\boldsymbol{\pi}}\boldsymbol{\xi})(I - \boldsymbol{\pi}\boldsymbol{\xi}')$, can be decomposed as follows:*

$$\boldsymbol{S}(\boldsymbol{\pi}, \boldsymbol{y}) = \boldsymbol{\Sigma}(\boldsymbol{\pi}, \boldsymbol{y}) + \boldsymbol{R}(\boldsymbol{\pi}, \boldsymbol{y}), \tag{3}$$

where $\boldsymbol{\Sigma}$ is a negative-definite symmetric matrix and $\boldsymbol{R}$ a matrix of rank no greater than S.

Proof : See Browning and Chiappori (1998).[8] ■

The intuition for this result is as follows. The matrix $\boldsymbol{S}(\boldsymbol{\pi}, \boldsymbol{y})$ in proposition 1 is, in fact, a *"pseudo" Slutsky matrix*. This is because the elements of the matrix $\boldsymbol{S}(\boldsymbol{\pi}, \boldsymbol{y})$ no longer represent the price effects on demand for a given level of household utility, as in the unitary model. In the collective framework, the price variation generates two effects. When the utility level and Pareto weights are given, a variation in prices changes the household's choices. This change satisfies the symmetry and negativity of the matrix of price effects while shifting the Pareto frontier. This effect corresponds to $\boldsymbol{\Sigma}(\boldsymbol{\pi}, \boldsymbol{y})$ in equation (3). However, this price variation also has an impact on the negotiating power of the household members through its effect on the Pareto weights and therefore on the chosen position on the new Pareto frontier. This effect corresponds to $\boldsymbol{R}(\boldsymbol{\pi}, \boldsymbol{y})$. The rank of this matrix is no greater than the number of Pareto weights (= S), owing to the fact that the price effects only influence consumer choices over the weights $\boldsymbol{\mu}$ (cf. equation (2)).

We may wonder whether this restriction is binding. Intuitively, we would expect that the fewer the decision makers and the greater the number of goods, the more this restriction will be binding. Thus, it is obvious that symmetry and negativity of the Slutsky matrix must obtain in the case of a household with a single individual (a Pareto weight of zero). Formally, Chiappori, Ekeland and Browning (1999) demonstrate that symmetry is only binding if $2(S+1) \leq N$, and negativity only if $S+1 < N$. Thus, symmetry is not binding in a classical labor supply model with three goods ($N = 3$) and two decision makers ($S = 1$).

The collective model's representation of households comprised of several individuals underlines the fact that violation of the traditional Slutsky conditions can be attributed to the omission of the relative bargaining power on the decision process. Furthermore, if the representation is valid, it also shows how and why this violation occurs. Empirically a test for the restriction of Proposition 1 reduces to testing for the following restriction:

Proposition 2 *Let $\boldsymbol{M}(\boldsymbol{\pi}, \boldsymbol{y}) = \boldsymbol{S}(\boldsymbol{\pi}, \boldsymbol{y}) - \boldsymbol{S}(\boldsymbol{\pi}, \boldsymbol{y})'$. Then the rank of the antisymmetric matrix $\boldsymbol{M}(\boldsymbol{\pi}, \boldsymbol{y})$ is no greater than $2S$.*

[8] Chiappori and Ekeland (2005) also demonstrate the converse proposition: If the condition SR(S) is valid and (given certain hypotheses) reasonable, then there exist Pareto weights and individual utility functions such that $\xi(\pi, y)$ solves the program (P).

Proof : See Chiappori and Ekeland (2005). ■

This proposition indicates that the implementation of the restrictions imposed by Proposition 1 leads to a rank test on an observable matrix.

2.2 Tests on the distribution factors

In the collective approach, the presence of distribution factors in the demand functions also generates testable restrictions on the household's decisions. The recent literature has presented two types of tests for these restrictions. The first applies to unconditional demands ($\boldsymbol{\xi}(\boldsymbol{\pi}, \boldsymbol{y})$), while the second is based upon so-called conditional demands. These are obtained as follows: Starting with a subset of the unconditional demand functions, we find the related inverse demand functions with respect to as many distribution factors, provided the inverse functions exist. Next we substitute these inverse functions into the remaining unconditional demand functions to obtain the conditional demand functions.

We begin by deriving a proposition related to the unconditional demands. Let $\boldsymbol{\xi}(\boldsymbol{\pi}, \boldsymbol{y})$ be a system of demand functions satisfying the condition SR(S), and $\boldsymbol{Y} = D_{\boldsymbol{y}}\boldsymbol{\xi}$ a matrix, the (i, k)-th element of which is $\frac{\partial \xi_i}{\partial y_k}$. We have the following proposition.

Proposition 3 *(Chiappori and Ekeland, 2005) Assume that the number of distribution factors and the number of goods exceed the number of Pareto weights (i.e., $K \geq S$ and $N \geq S$). Then we have rank $\boldsymbol{Y} \leq S$.*

Proof : The proof is straightforward. Since at the optimum we have: $\boldsymbol{\xi}(\boldsymbol{\pi}, \boldsymbol{y}) = \tilde{\boldsymbol{\xi}}(\boldsymbol{\pi}, \boldsymbol{\mu}(\pi, \boldsymbol{y}))$, the matrix of the effects of the distribution factors on the demand functions is given by $\boldsymbol{Y} = D_{\boldsymbol{y}}\xi(\boldsymbol{\pi}, \boldsymbol{y}) = D_{\boldsymbol{\mu}}\tilde{\xi}(\boldsymbol{\pi}, \boldsymbol{\mu})D_y\boldsymbol{\mu}$. Because the dimension of the matrix $D_y\boldsymbol{\mu}$ is $S \times K$, its rank can be no greater than S. Consequently, rank $\boldsymbol{Y} \leq S$. ■

Proposition 3 stipulates that if there are more than S distribution factors, their effects on the demands must be linearly dependent. This is a generalization of the result obtained by Bourguignon *et al.*, (1995) for the case of a household with two decision makers ($S = 1$). Thus, the distribution factors have proportional effects on all demands. Formally, if $S = 1$ and $K \geq 2$, then the expression $\frac{\partial \xi_i}{\partial y_k} / \frac{\partial \xi_i}{\partial y_l}$ is the same for all i.

The intuition underlying this result is as follows. The demand system depends on the Pareto weights of the first S decision makers relative to that of the $(S + 1)$-st. If there are fewer decision makers than there are distribution factors, then the effects these factors have on the demands must necessarily be (locally) linearly dependent. Indeed, by definition they only impact on demands over the Pareto weights.[9]

[9] A result from Browning and Chiappori (1998) to the effect that there is a link between price effects and the effects of the distribution factors is also generalized to the case of several decision makers by Chiappori and Ekeland (2005). This link springs from the fact that, in the collective model, a part of the price effects (that part that violates the symmetry and negativity of the Slutsky matrix) and the effects of distribution factors act on the demands through the Pareto weights. This property, which is in theory falsifiable, is not tested in this paper.

Although the test in Proposition 3 is for unconditional demands, the tests we are now proposing apply to conditional demands. These tests draw on the work of Bourguignon *et al.*, (1995), Dauphin and Fortin (2001), and Dauphin (2003). To simplify the notation, we suppress prices (assumed fixed) hereafter.

Consider the partitions $\boldsymbol{\xi} \equiv \left[\boldsymbol{\xi}_1', \boldsymbol{\xi}_2'\right]'$ and $\boldsymbol{y} \equiv [\boldsymbol{y}_1', \boldsymbol{y}_2']'$ of the vectors $\boldsymbol{\xi}$ and $\boldsymbol{y}$ with dim $\boldsymbol{\xi}_1 =$ dim $\boldsymbol{y}_1 = J$. The demand system (2) can then be written as:

$$\boldsymbol{\xi}_1 = \boldsymbol{\xi}_1(\boldsymbol{y}_1, \boldsymbol{y}_2) \equiv \tilde{\boldsymbol{\xi}}_1\left(\boldsymbol{\mu}\left(\boldsymbol{y}_1, \boldsymbol{y}_2\right)\right), \tag{4}$$

$$\boldsymbol{\xi}_2 = \tilde{\boldsymbol{\xi}}_2(\boldsymbol{y}_1, \boldsymbol{y}_2) \equiv \tilde{\boldsymbol{\xi}}_2\left(\boldsymbol{\mu}\left(\boldsymbol{y}_1, \boldsymbol{y}_2\right)\right). \tag{5}$$

Lemma 1 *Let $\boldsymbol{\xi}_1(\boldsymbol{y})$ be differentiable and the matrix $D_{\boldsymbol{y}_1}\boldsymbol{\xi}_1(\boldsymbol{y})$ nonsingular. Then, conditional on $\boldsymbol{\xi}_1 = \boldsymbol{\xi}_1(\boldsymbol{y}_1, \boldsymbol{y}_2)$, there exists a differentiable and unique vector function $\boldsymbol{y}_1 = \boldsymbol{y}_1(\boldsymbol{\xi}_1, \boldsymbol{y}_2)$ that (locally) solves (3) for $\boldsymbol{y}_1$ and that satisfies:*

$$\boldsymbol{\xi}_1 = \boldsymbol{\xi}_1\left(\boldsymbol{y}_1(\boldsymbol{\xi}_1, \boldsymbol{y}_2), \boldsymbol{y}_2\right) = \tilde{\boldsymbol{\xi}}_1\left(\boldsymbol{\mu}(\boldsymbol{y}_1(\boldsymbol{\xi}_1, \boldsymbol{y}_2)), \boldsymbol{y}_2\right) \tag{6}$$

Proof : Application of the implicit function theorem. ■

Under the conditions of Lemma 1, we can define a function $\bar{\boldsymbol{\xi}}_2 : \mathbb{R}^K \rightarrow \mathbb{R}^{N-J}$, such that:

$$\bar{\boldsymbol{\xi}}_2(\boldsymbol{\xi}_1, \boldsymbol{y}_2) = \boldsymbol{\xi}_2\left(\boldsymbol{y}_1(\boldsymbol{\xi}_1, \boldsymbol{y}_2), \boldsymbol{y}_2\right) = \tilde{\boldsymbol{\xi}}_2\left\{\boldsymbol{\mu}\left(\boldsymbol{y}_1(\boldsymbol{\xi}_1, \boldsymbol{y}_2), \boldsymbol{y}_2\right)\right\}, \tag{7}$$

where $\bar{\boldsymbol{\xi}}_2(\boldsymbol{\xi}_1, \boldsymbol{y}_2)$ is a subsystem of demands for the $(N-J)$ last goods, defined conditional on the demands for the first J goods, $\boldsymbol{\xi}_1$.

Proposition 4 *Let the assumptions of Lemma 1 be satisfied and $\boldsymbol{\mu}(\boldsymbol{y})$ and $\tilde{\boldsymbol{\xi}}\left[\boldsymbol{\mu}(\boldsymbol{y})\right]$ be differentiable. Also assume that $K \geq S$ and $N \geq S$. Then, for $J = 1, \ldots, S$, the rank of the matrix $D_{\boldsymbol{y_2}}\bar{\boldsymbol{\xi}}_2(\boldsymbol{\xi}_1, \boldsymbol{y}_2)$ is equal to, or less than, $(S-J)$.*

Proof : See the Appendix. ■

We begin by noting that this result has already been established for $J = S$ by Dauphin and Fortin (2001). The intuition underlying Proposition 4 is as follows. We know that the unconditional demand system depends on the S Pareto weights. To maintain constant any conditional demand, the distribution factor upon which it is conditioned must compensate for variations in the other distribution factors, implying that at least one of the relative weights affecting this demand must be linearly dependent on the other relative weights. Consequently, the subsystem of conditional demands will depend on at most $(S-J)$ relative weights, and the rank of the matrix of the effects of the distribution factors will be at most $(S-J)$.

It is important to note that Propositions 3 and 4 are equivalent provided Lemma 1 is satisfied. More precisely, when the result in Proposition 3 does (does not) obtain, the result in Proposition 4 does (does not) obtain, and vice versa. This equivalence is easy to see. When $rank[D_{\mathbf{y}_1}\boldsymbol{\xi}_1] = J$, we know immediately that the minimum rank of $D_{\mathbf{y}}\boldsymbol{\xi}$ is J. Now, if $rank[D_{\mathbf{y}_2}\bar{\boldsymbol{\xi}}_2] \leq S-J$, this is because $\mathbf{y}_2$ generates as many as $S-J$ linearly independent effects on $\bar{\boldsymbol{\xi}}_2$. Thus, the vector $\mathbf{y}$ can generate a total of $(S-J)+J$ linearly independent effects on $\boldsymbol{\xi}$, and so $rank[D_{\mathbf{y}}\boldsymbol{\xi}] \leq S$. Conversely, if $rank[D_{\mathbf{y}}\boldsymbol{\xi}] \leq S$, this is because $\mathbf{y}_2$ generates as many as S linearly independent effects on $\boldsymbol{\xi}$. If we condition $\boldsymbol{\xi}_2$ on $\boldsymbol{\xi}_1$ with $rank[D_{\mathbf{y}_1}\boldsymbol{\xi}_1] = J$, no more than $S - J$ linearly independent effects can remain.

For these two results to constitute a test for collective rationality, it is necessary that $K \geq S + 1$ and $N > S + 1$. If $K = N = S$, the dimension of the matrix $D_{\mathbf{y}}\boldsymbol{\xi}$ will be S and thus necessarily of rank no greater than S. If $K = N = S + 1$, the dimension of the matrix $D_{\mathbf{y}}\boldsymbol{\xi}$ will be $S + 1$ and its rank will still be no greater than S owing to Walras' law.

Though Propositions 3 and 4 are equivalent from a theoretical perspective, the statistical tests that derive from them need not yield identical results, especially in small samples. For this reason, it is best to test for collective rationality using on the basis of both propositions. That is what we do in the empirical section of this paper.

The following corollary is useful because it allows the number of decision makers in a household (or a group in general) to be determined when collective rationality can reasonably be assumed to hold.

Corollary 1 *(Dauphin and Fortin, 2001) Assume that the decision process is collectively rational. Also let the rank of* $D_{\boldsymbol{\mu}}\tilde{\xi}_2\left(\boldsymbol{\mu}(\boldsymbol{y})\right) = S$ *for all* $J < S$*. Given the assumptions of Proposition 4, the number of decision makers in the household corresponds to the smallest number of goods on which the demand functions must be conditioned for Proposition 4 to be satisfied for* $J = S$*, i.e.* $D_{\boldsymbol{y}_2}\bar{\boldsymbol{\xi}}_2(\boldsymbol{\xi}_1, \boldsymbol{y}_2) = \mathbf{0}$*, plus 1.*

Proof : See the Appendix. ■

We will present the results generated by application of this corollary in the next section.

3 Empirical Tests

Sections 2.1 and 2.2 showed that collective rationality generates falsifiable restrictions concerning the impact of prices and distribution factors on households' aggregate consumption choices. The theoretical results we presented also demonstrated that these restrictions vary with the number of decision makers in the household. In Section 3.1 we use a series of cross-sectional data drawn from the British Family Expenditure Survey (FES) to apply this type of test to the case of households that may contain three decision makers.

3.1 Analysis of British households

We use data drawn from the FES covering the period 1982–1993. The survey contains a broad array of information on household expenditures on durable and nondurable goods, on the incomes and labor supplies of the various adult members of the household, and on their socio-economic characteristics. From the annual surveys we selected a subsample of 2745 families comprising three adults, *i.e.* a married couple and a child aged 16 or over. To make the sample more homogeneous, we selected families with no young children. We also excluded households in which one of the two parents was not active on the labor market, or nearing retirement (men over 65 or women over 60), as well as households residing in Northern Ireland. Finally, we excluded households in which one of the three decision makers was not earning any income.

While we are interested in household consumption, we can only observe consumption expenditure. This distinction is important on a conceptual level. Expenditure on a nondurable good at time t is a good approximation for its consumption at that time. However, durable goods provide a flow of services that are consumed over a period of time. Consequently, expenditure on a durable good is an unsatisfactory measure of its consumption. We assume that the distinction between a nondurable good and a durable good can be made unambiguously. We also assume weak separability between the consumption of durable and nondurable goods. Thus, choices concerning nondurable goods depend on total expenditure on these goods, *i.e.* the household's total income net of expenditures on durable goods. The assumption of separability, while restrictive, is common in the literature (cf. Banks *et al.*, 1997). Furthermore, note that we condition our estimation on home and car ownership to allow us to test a certain form of separability between durable and nondurable goods.

The demand system we estimate comprises 11 categories of nondurable goods: food, restaurant meals, alcohol, tobacco, services, leisure, heating, transportation, clothing, recreational goods, and personal goods. In the terms of our theory, our sample is thus characterized by $S = 2$ (two Pareto weights) and $N = 11$. Prices are measured monthly at the county level, yielding 144 different prices for each good.

Testing Proposition 3 in this context requires observing at least three distribution factors. Given the structure of the chosen households, and in light of the fact that the demand system is conditioned on total expenditure on nondurable goods, we can construct three distribution factors from individual incomes. Following Browning and Chiappori (1998), we use the log of one spouse's gross income (the husband in this case), the difference between the logs of the two spouses' incomes [log (wife's income) – log(husband's income)], and the difference between the logs of the child's and the father's income [log(child's income) – log(father's income)].[10] Of course, these distribution factors need to be used cautiously since their validity depends on the assumption of separability. Nonetheless, they may have a significant impact on the decision-making process at the household level for given total expenditures on nondurable goods. Other factors could eventually be included in the analysis (*e.g.*, sex ratio, divorce law). For the moment, the analysis uses only three distribution factors, given the unavailability of data on other factors.

Table 1 presents descriptive statistics from our sample. These statistics are compiled for all years from 1982 to 1993. Since we omit durables, it is not surprising that the largest shares relate to food, recreational goods, and clothing. Moreover, the distribution factors suggest a significant gap between the spouses's income on one hand, and between the father's and the child's income, on the other. Other variables in the table reveal that the majority of households have a car (83.2%) and about half own a house (48.9%). Finally, the spouses' education levels are similar, while the children are slightly less educated, presumably because of their age.

[10]Browning and Chiappori (1998) ignore this last distribution factor since they assume in their analysis that only the spouses are decision makers in the household.

3.1.1 The empirical model

To implement the empirical tests, we estimate a Quadratic Almost Ideal Demand System (QAIDS) as proposed by Banks *et al.*, (1997) and used by Browning and Chiappori (1998). The QAIDS system has the advantage of allowing a flexible functional form that captures nonlinearity in the Engel curves. It has been validated empirically several times (cf. Banks *et al.*, 1997).

The budget shares are written as:

$$w = \boldsymbol{\alpha} + \boldsymbol{\Theta y} + \boldsymbol{\Gamma p} + \boldsymbol{\beta}\left(\ln(m) - a(\boldsymbol{p})\right) + \boldsymbol{\lambda}\frac{\left(\ln(m) - a(\boldsymbol{p})\right)^2}{b(\boldsymbol{p})} + \boldsymbol{v}, \tag{8}$$

where $\boldsymbol{\alpha}, \boldsymbol{\beta}$ and $\boldsymbol{\lambda}$ are $(N \times 1)$ vectors of parameters, $\boldsymbol{\Theta}$ and $\boldsymbol{\Gamma}$ are $(N \times K)$ and $(N \times N)$ matrices of parameters, respectively, $\boldsymbol{y}$ is a $(K \times 1)$ vector of distribution factors, $\boldsymbol{p}$ is an $(N \times 1)$ vector of log prices, $\ln(m)$ is the log of the household's total expenditure on nondurable goods, and $\boldsymbol{v}$ is a vector of error terms. The price indexes $a(\boldsymbol{p})$ and $b(\boldsymbol{p})$ are defined by:

$$a(\boldsymbol{p}) = \alpha_0 + \boldsymbol{\alpha' p} + \frac{1}{2}\boldsymbol{p'\Gamma p} \tag{9}$$

$$b(\boldsymbol{p}) = exp(\boldsymbol{\beta' p}) \tag{10}$$

Additivity implies that $\boldsymbol{\alpha' e} = \mathbf{1}$, $\boldsymbol{\Theta' e} = \mathbf{0}$ and $\boldsymbol{\beta' e} = \boldsymbol{\lambda' e} = \boldsymbol{\Gamma e} = \mathbf{0}$, where $\boldsymbol{e}$ is an N-dimensional unit vector. Homogeneity implies that $\boldsymbol{\Gamma' e} = \mathbf{0}$. In practice, additivity necessarily obtains owing to the construction of the data in terms of budget shares. Thus, we estimate a system of 10 equations rather than 11 by eliminating an arbitrarily chosen equation from the system. The parameters of the omitted equation are obtained by substituting it into the budget constraint. To simplify notation, we let $N = 10$ in the following. We impose homogeneity by substituting relative prices for absolute prices (dividing them all by the price of heating—the reference price).

In equation (8), distribution factors are introduced so as to only affect the constants in the share equations. This procedure has two advantages. First, the inverse demands (when they exist) have the same functional form as the unconditional demands, provided that the distribution factors with respect to which they are inverted are replaced on the right by expenditure shares. Second, and with the same proviso, conditional demands on the remaining shares have the same functional form as the corresponding unconditional demands. These two properties greatly facilitate application of the tests.

The Slutsky matrix is given by:

$$\boldsymbol{S} = \boldsymbol{\Gamma} - \frac{1}{2}\left(\boldsymbol{\beta} + 2\boldsymbol{\lambda}\frac{\widetilde{m}}{b(\boldsymbol{p})}\right)\boldsymbol{p}'(\boldsymbol{\Gamma} - \boldsymbol{\Gamma}') + \widetilde{m}\left\{\boldsymbol{\beta\beta}' + \frac{\widetilde{m}}{b(\boldsymbol{p})}(\boldsymbol{\lambda\beta}' + \boldsymbol{\beta\lambda}') + \left(\frac{\widetilde{m}}{b(\boldsymbol{p})}\right)^2 \boldsymbol{\lambda\lambda}'\right\}, \tag{11}$$

with $\widetilde{m} = \ln(m) - a(\boldsymbol{p})$.

So far, we have ignored preference variables which take into account observable individual heterogeneity. However, in the empirical specification of the model we estimate the demand system with a vector $\boldsymbol{z}$ of socio-demographic characteristics incorporated via the functions $a(\boldsymbol{p})$ and $b(\boldsymbol{p})$. More precisely, we write:

$$a(\boldsymbol{p}, \boldsymbol{z}) = \alpha_0 + \boldsymbol{\alpha}(\boldsymbol{z})'\boldsymbol{p} + \frac{1}{2}\boldsymbol{p}'\Gamma\boldsymbol{p}, \text{ and} \tag{12}$$

$$b(\boldsymbol{p}, \boldsymbol{z}) = \exp\left(\boldsymbol{\beta}(\boldsymbol{z})'\boldsymbol{p}\right), \tag{13}$$

where the functions $\boldsymbol{\alpha}(\boldsymbol{z})$ and $\boldsymbol{\beta}(\boldsymbol{z})$ are linear in $\boldsymbol{z}$. The vector $\boldsymbol{z}$ includes a series of dummy variables (nine regional variables, three seasonal dummies, car and home ownership). Preliminary estimations revealed that the variables for education and age were never significant, possibly owing to the homogeneity of the sample. Given the QAIDS specification, Proposition 1 reduces to the following proposition.

Proposition 5 *The matrix S satisfies the restriction SR(S) if and only if the matrix Γ satisfies the restriction SR(S).*

Proof : This is an immediate generalization of Proposition 10 from Browning and Chiappori (1998). ■

The result in Proposition 5, combined with that in Proposition 2, reduces the empirical test for the restriction SR(S) to a test of the following hypothesis:

$$\boldsymbol{H_0} : \text{rank (M)} = \text{rank}(\Gamma - \Gamma^{'}) \leq 2S.$$

The linearity of the QAIDS demand system is conditional on the terms $a(\boldsymbol{p})$ and $b(\boldsymbol{p})$. Consequently, it can be directly estimated using iterated ordinary least squares as proposed by Blundell and Robin (1999). Basically, this approach consists in estimating the ordinary least squares system after having replaced the coefficients in $a(\boldsymbol{p})$ and $b(\boldsymbol{p})$ by some initial values. We subsequently iterate until convergence, accounting for the fact that these two expressions depend on the system's estimated coefficients (except α_0, which is held constant like in Browning and Chiappori (1998)).

To account for the possibility that the log of total expenditure on nondurable goods is endogenous, we add the residuals of an auxiliary regression of the log of these expenditures on a set of instruments into the QAIDS specification in (8).[11] Thus, the error term $\boldsymbol{v}$ can be written as the orthogonal decomposition

$$\boldsymbol{v} = \rho u + \epsilon. \tag{14}$$

Testing $\rho = 0$ is equivalent to a test for the exogeneity of the log of total expenditure on nondurables. The instruments used are all the explanatory variables, a term for a linear trend, the log of the household's net income and its square, and a general price index. [12]

3.1.2 Estimation of the demand system

The parameters of the unconditional demand system are presented in Table 2. The three distribution factors have a significant impact in several of the demand equations. This

[11] After a few experiments, and following Banks *et al.*, we chose not to include a residual generated by a regression of the square of expenditures on the instruments as an additional variable.

[12] The results of the instrumental regressions are not presented for the sake of brevity. They can be provided upon request.

is particularly the case in the demand for food, tobacco, leisure, and recreational goods. Furthermore, most of the coefficients of the relative prices are statistically significant. [13] All the coefficients of prices are negative, except for transportation.

Overall, ownership of a car or a house has little impact on the consumption of nondurables, since few of the parameters are statistically significant. This result is consistent with the separability assumption between durable and nondurable goods. Conversely, our results reveal that the quadratic term for the log of total expenditure is significant in all demands except for food. This result is consistent with that obtained by Banks *et al.* (1997).

The test for exogeneity presents the t-statistic of the parameter associated with the regression of total expenditure on a set of instruments. As indicated, exogeneity is rejected for three of the ten demand functions. Finally, according to the last line of Table 2, the instruments pass the joint test for validity and overidentification.

3.1.3 Tests for the collective model

The parameters in Table 2 can be directly used to test the collective model. The various propositions presented in Sections 2.1 and 2.2 are tested in turn.

1. *Tests for the price effect*

   (a) *Test of SR(S)*
   As mentioned above, testing the SR(S) restriction is equivalent to testing whether the rank of the (10×10) antisymmetric matrix $M = \Gamma - \Gamma'$ is less than or equal to $4 = 2 \times (3 - 1)$. To implement this test, we follow the procedure proposed by Kleibergen and Paap (2003)[14] based on the decomposition of the matrix to be tested into singular values.[15] The test is based on the following hypothesis:

$$\begin{aligned} \boldsymbol{H_0} &: \; rank(M) = q \\ \boldsymbol{H_1} &: \; rank(M) \geq q \end{aligned}$$

   When $q = 0$, the test amounts to the testing symmetry of the price effects. The table below reveals that symmetry is strongly rejected, since the statistic is $\chi^2_{(100)} = 349.39$. Successively increasing the value of q indicates that only the assumption that $rank(M) = 4$ cannot be rejected. This result is consistent with the collective model and, as far as we know, constitutes the first test to validate the hypothesis of collective rationality when there are more than two decision makers in the household and using data from a developed country.

[13] Preliminary estimates revealed that the assumption of homogeneity cannot be rejected.

[14] The literature contains several other tests for the rank of a matrix, including those proposed by Robin and Smith (2000), Craag and Donald (1997), and Gill and Lewbel (1992). However, the two last are only applicable when the variance-covariance of the matrix to be tested is nonsingular. In addition, the test statistic proposed by Robin and Smith (2000) does not follow a standard distribution, making the test procedure difficult to implement. See Kleibergen and Paap (2003) for details.

[15] The singular values of a matrix A are the square roots of the eigenvalues of $A'A$.

TEST OF PROP. SR(S)

Rank q	Test[†]	P-value
0	349.39	0.000
1	365.90	0.000
2	204.03	0.000
3	140.00	0.000
4	42.50	0.211

[†] Test $\backsim \chi^2((10-q)(10-q))$

2. *Tests for the effects of the distribution factors*

   (a) *Test of Proposition 3*
   Proposition 3 states that the rank of the 3×10 matrix of the effects of the distribution factors cannot exceed 2 in our framework. These test statistics are derived using the same procedure as we used for testing the restriction SR(S). They are given in the table below. Our results reveal that the test of collective rationality based on the distribution factors cannot be rejected either.

TEST FOR PROP. 3

Rank q	Test[†]	P-value
0	108.10	0.000
1	42.27	0.001
2	0.84	0.099

[†] Test $\backsim \chi^2((10-q)(3-q))$

   (b) *Test of Lemma 1*
   Lemma 1 yields the conditions for inverting the demand system to define a conditional demand system. With 10 demand equations and 3 distribution factors, there are as many as 135 possibilities for inversion with respect to 2 distribution factors. For the sake of brevity, we only report two inversion possibilities that satisfy the conditions of Lemma 1. The demand equations are given by

$$\begin{aligned} Food &= \alpha_1 FD1 + \alpha_2 FD2 + \alpha_3 FD3 + \text{ ....} \\ Tobacco &= \beta_1 FD1 + \beta_2 FD2 + \beta_3 FD3 + \text{ ....} \\ Recreational\ goods &= \gamma_1 FD1 + \gamma_2 FD2 + \gamma_3 FD3 + \text{ ....} \end{aligned}$$

   The results of the rank test for the following matrices

$$A = \begin{pmatrix} \alpha_1 & \alpha_3 \\ \beta_1 & \beta_3 \end{pmatrix} \text{ and } B = \begin{pmatrix} \alpha_1 & \alpha_3 \\ \gamma_1 & \gamma_3 \end{pmatrix}$$

are

TEST FOR NONSINGULARITY

Matrix A

Rank q	Test	P-value
0	41.39	0.000
1	5.03	0.024

Matrix B

Rank q	Test	P-value
0	38.62	0.000
1	6.75	0.009

Test $\backsim \chi^2((2-q)(2-q))$

These results show that we can use a demand system that is conditioned on the budget shares of food and tobacco, or conditioned on the shares of food and leisure, by inverting these shares on the distribution factors FD1 and FD3, respectively.

(c) *Test of Proposition 4*

Proposition 4 states that, in the case of three decision makers, the rank of the matrix of the effects of the distribution factors on the conditional demand is no greater than 2 minus the number of demands having been used to condition the system. Table 3 presents the estimation results of the demand system conditional on the share of food.[16] First note that in order to account for the possibility that the conditioning good is endogenous, the demand system includes the residual of the auxiliary regression of this good on a set of instruments. An immediate instrument for the conditioning good is the distribution factor with respect to which we inverted the demand equation (FD1, log of the husband's gross income). The two last lines of Table 3 present the test for exogeneity of the conditioning good and the joint test for overidentification and the validity of the instruments used, respectively. We observe that exogeneity is only rejected for total expenditure and the share of food in the case of three demands (alcohol, clothing, and transportation). Furthermore, joint tests for overidentification and the validity of the instruments are satisfied at the usual levels.

In this demand system, the marginal effects of the distribution factors yield a is 9×2 matrix whose rank can not be greater than 1 according to Proposition 4. The results of the rank test are derived with the same procedure as before. They are given in the following table.

[16] Of course, other specifications are possible. In the interest of brevity, we present the results of the demand system estimation conditional on the share of food.

TEST OF PROP. 4

Rank q	Test	P-value
0	70.50	0.000
1	9.24	0.322

Test $\backsim \chi^2((9-q)(2-q))$

Once again, the test statistics do not reject the validity of the restrictions imposed by the collective model.

(d) *Test of Corollary 1*

Table 4 presents the results of the demand system estimation conditional on the shares of food and tobacco. As in the case of Table **??**, tests for the exogeneity of total expenditures and the share of food are only rejected for three demands. Conversely, the exogeneity of leisure is not rejected in any case. For all the demand functions in the Table, the appropriate tests do not lead us to reject the hypothesis of overidentification or the validity of the instruments.

Corollary 1 states that, conditional on Proposition 4, the number of decision makers corresponds to the minimum number of goods on which the demand functions must be conditioned for the effect of the remaining distribution factors to be nil, plus 1. The effect of the remaining distribution factors in the demand system is not significant in any of the demand equations. This leads us to the conclusion that there are, in fact, 2 (distribution factors) + 1 = 3 decision makers in our sample of British households.

4 Conclusion

To the best of our knowledge, this paper presents the first study applied to a developed country (Great Britain) that seeks to test the assumption of Pareto efficiency (or collective rationality) in the framework of households with more than two decision makers. In particular, our study allows the analysis of consumption choices of couples living with adult children or the elderly. We first present an overview of the tests that have been developed in the recent literature on collective rationality. The analysis uses a theoretical framework in which the consumed goods may be private or public and may feature externalities. These tests are based on the impact of prices and distribution factors on households' aggregate demand. More formally, they boil down to testing the rank of a matrix that captures the compensated price effects and a matrix of the the marginal effects of the distribution factors. As to the price effects, the underlying idea is that, in the collective model, prices not only influence the household's decisions through traditional substitution and income effects, but also through their impact on the weights of decision makers in the household's utility function. Since the number of (Pareto) weights equals the number of decision makers (minus one), restrictions arise on the rank of the matrix of compensated price effects. When they are binding, these restrictions can be tested empirically. A similar argument applies to the marginal effects of the distribution factors, since they can only affect the Pareto weights. Moreover, we have developed a new test dealing with the effects of distribution factors on conditional demands. This test, which is equivalent to the previous test under certain conditions, is likely much easier to implement. Finally, we present a test that allows to determine the number of decision makers in the household under the assumption of collective rationality.

All of the above tests are used to investigate the collective rationality of households drawn from a series of British Family Expenditure Surveys covering the period 1982–1993. The econometric model focuses on consumption expenditures of couples with a single child over 16 years old living at home. Our rank tests lead us to reject collective rationality when only one or two decision makers are assumed to choose the observed consumption bundles. On the other hand, collective rationality can not be rejected when the household is assumed to be comprised of three decision makers. Our analysis thus leads us to the conclusion that there are three decision makers in our sample.

Our results have important implications for the analysis of intrafamily welfare. They specifically reveal that it may be incorrect to assume that there are no more than two decision makers within the household. This latter hypothesis, which has never before been tested and is assumed accurate in virtually all empirical studies of consumption, can thus lead to biased estimates of household demand functions. It can also lead to unwarranted inferences as to the impact of social policy (*e.g.* transfer programs) on intrahousehold welfare. More generally, household choices vary with the number of decision makers, and it is essential to account for this in any analysis. A natural extension to our study would be to determine at what age a child becomes a decision maker within the household.

Appendix

Proof of Proposition 4:

Differentiating equations (6) and (7) with respect to $\boldsymbol{y}_2$, and evaluating at the point $\boldsymbol{y}_2$, yields:

$$\begin{aligned} \mathbf{0} &= D_{\boldsymbol{\mu}} \tilde{\boldsymbol{\xi}}_1 \left(\boldsymbol{\mu}(\boldsymbol{y})\right) D_{\boldsymbol{y}_2} \boldsymbol{\mu} \left(\tilde{\boldsymbol{y}}_1(\boldsymbol{\xi}_1, \boldsymbol{y}_2), \boldsymbol{y}_2\right), & (15) \\ D_{\boldsymbol{y}_2} \bar{\boldsymbol{\xi}}_2(\boldsymbol{\xi}_1, \boldsymbol{y}_2) &\equiv D_{\boldsymbol{\mu}} \boldsymbol{\xi}_2 \left(\boldsymbol{\mu}(\boldsymbol{y})\right) D_{\boldsymbol{y}_2} \boldsymbol{\mu} \left(\boldsymbol{y}_1(\boldsymbol{\xi}_1, \boldsymbol{y}_2), \boldsymbol{y}_2\right). & (16) \end{aligned}$$

Under the conditions of Lemma 1 we know that the matrix $D_{\boldsymbol{y}_1} \boldsymbol{\xi}_1(\boldsymbol{y})$ is nonsingular and that the rank of the $J \times S$-dimensional matrix $D_{\boldsymbol{\mu}} \tilde{\boldsymbol{\xi}}_1 \left(\boldsymbol{\mu}(\boldsymbol{y})\right)$ is J given that $D_{\boldsymbol{y}_1} \boldsymbol{\xi}_1(\boldsymbol{y}) = D_{\boldsymbol{\mu}} \tilde{\boldsymbol{\xi}}_1 \left(\boldsymbol{\mu}(\boldsymbol{y})\right) D_{\boldsymbol{y}_2} \boldsymbol{\mu}(\boldsymbol{y})$. Thus, we can arbitrarily partition this latter into a non-singular $J \times J$ matrix and a $J \times (S-J)$ matrix as follows:
$\left(D_{\boldsymbol{\mu}_J} \tilde{\boldsymbol{\xi}}_1 \left(\boldsymbol{\mu}(\boldsymbol{y})\right) \quad D_{\boldsymbol{\mu}_{S-J}} \tilde{\boldsymbol{\xi}}_1 \left(\boldsymbol{\mu}(\boldsymbol{y})\right)\right)$, where $\boldsymbol{\mu}_J \equiv (\mu_1, \ldots, \mu_J)$ and $\boldsymbol{\mu}_{S-J}$ is the complement of $\boldsymbol{\mu}_J$. Therefore, equation (15) can be rewritten as:

$$D_{\boldsymbol{y}_2} \boldsymbol{\mu}_J \left(\boldsymbol{y}_1(\boldsymbol{\xi}_1, \boldsymbol{y}_2), \boldsymbol{y}_2\right) = -\left\{D_{\boldsymbol{\mu}_J} \tilde{\boldsymbol{\xi}}_1 \left(\boldsymbol{\mu}(\boldsymbol{y})\right)\right\}^{-1} D_{\boldsymbol{\mu}_{S-J}} \tilde{\boldsymbol{\xi}}_1 \left(\boldsymbol{\mu}(\boldsymbol{y})\right) \left\{D_{\boldsymbol{y}_2} \boldsymbol{\mu}_{S-J} \left(\boldsymbol{y}_1(\boldsymbol{\xi}_1, \boldsymbol{y}_2), \boldsymbol{y}_2\right)\right\}.$$

Substituting this result into equation (16) yields:

$$D_{\boldsymbol{y}_2} \bar{\boldsymbol{\xi}}_2(\boldsymbol{\xi}_1, \boldsymbol{y}_2) = A \left\{D_{\boldsymbol{y}_2} \boldsymbol{\mu}_{S-J} \left(\boldsymbol{y}_1(\boldsymbol{\xi}_1, \boldsymbol{y}_2), \boldsymbol{y}_2\right)\right\},$$

where $A \equiv \left(D_{\boldsymbol{\mu}_{S-J}} \tilde{\boldsymbol{\xi}}_2 \left(\boldsymbol{\mu}(\boldsymbol{y})\right) - D_{\boldsymbol{\mu}_J} \tilde{\boldsymbol{\xi}}_2 \left(\boldsymbol{\mu}(\boldsymbol{y})\right) \left\{D_{\boldsymbol{\mu}_J} \tilde{\boldsymbol{\xi}}_1 \left(\boldsymbol{\mu}(\boldsymbol{y})\right)\right\}^{-1} D_{\boldsymbol{\mu}_{S-J}} \tilde{\boldsymbol{\xi}}_1(\boldsymbol{\mu}(\boldsymbol{y}))\right)$.
Since the $(S-J) \times (K-J)$-dimensional matrix, $D_{\boldsymbol{y}_2} \boldsymbol{\mu}_{S-J}(\boldsymbol{y}_1(\boldsymbol{\xi}_1, \boldsymbol{y}_2), \boldsymbol{y}_2)$ is at most of rank $S-J$, the rank of $D_{\boldsymbol{y}_2} \bar{\boldsymbol{\xi}}_2(\boldsymbol{\xi}_1, \boldsymbol{y}_2)$ must also be smaller than or equal to $S-J$. **Q.E.D.**

Proof of Corollary 1:

According to Proposition 4, we know that $D_{\boldsymbol{y}_2} \bar{\boldsymbol{\xi}}_2(\boldsymbol{\xi}_1, \boldsymbol{y}_2) = \mathbf{0}$ for $J = S$. Furthermore, when $J < S$, there are an infinite number of nontrivial solutions to
$D_{\boldsymbol{y}_2} \boldsymbol{\mu} \left(\boldsymbol{y_1}(\boldsymbol{\xi_1}, \boldsymbol{y_2}), \boldsymbol{y_2}\right)$ that are consistent with the system of J equations in S variables $D_{\boldsymbol{\mu}} \tilde{\boldsymbol{\xi}}_1 \left(\boldsymbol{\mu}(y)\right) D_{\boldsymbol{y}2} \boldsymbol{\mu} \left(\boldsymbol{y_1}(\boldsymbol{\xi_1}, \boldsymbol{y_2}), \boldsymbol{y_2}\right) = \mathbf{0}$. Assuming that $(\boldsymbol{\xi_1}, \boldsymbol{y_2})$ does not correspond to the trivial solution, we have $D_{\boldsymbol{y}_2} \boldsymbol{\mu} \left(\boldsymbol{y_1}(\boldsymbol{\xi_1}, \boldsymbol{y_2}), \boldsymbol{y_2}\right) \neq \mathbf{0}$. Now, since $rank\ D_{\boldsymbol{\mu}} \tilde{\boldsymbol{\xi}}_2 \left(\boldsymbol{\mu}(y)\right) = S$ (implying that $N \geq S + J$), the only solution to the system of $N - J$ equations and S variables, $D_{\boldsymbol{\mu}} \tilde{\boldsymbol{\xi}}_2 \left(\boldsymbol{\mu}(y)\right) D_{\boldsymbol{y}2} \boldsymbol{\mu} \left(\boldsymbol{y_1}(\boldsymbol{\xi_1}, \boldsymbol{y_2}), \boldsymbol{y_2}\right) = \mathbf{0}$, is $D_{\boldsymbol{y}_2} \boldsymbol{\mu} \left(\boldsymbol{y_1}(\boldsymbol{\xi_1}, \boldsymbol{y_2}), \boldsymbol{y_2}\right) = \mathbf{0}$. Consequently, we must have $D_{\boldsymbol{y}_2} \bar{\boldsymbol{\xi}}_2(\boldsymbol{\xi}_1, \boldsymbol{y}_2) \neq \mathbf{0}$, since $D_{\boldsymbol{y}_2} \boldsymbol{\mu} \left(\boldsymbol{y_1}(\boldsymbol{\xi_1}, \boldsymbol{y_2}), \boldsymbol{y_2}\right) \neq \mathbf{0}$. **Q.E.D.**

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Table 1: Descriptive Statistics

Variable	**Mean**	**Standard-error**
Budget shares		
Food	0,287	0,168
Alcohol	0,063	0,086
Tobacco	0,056	0,078
Clothing	0,094	0,109
Leisure	0,036	0,072
Transportation	0,034	0,058
Service:Domestic phone service item	0,047	0,047
Restaurant	0,052	0,054
Personal goods (P.G.) Toiletries and other...	0,057	0,078
Recreational goods (R.G.)	0,120	0,095
Distribution Factors		
Ln(Gross income, man) (FD1)	5,131	0,934
Ln(Income, wife)-Ln(Income, husband) (FD2)	-1,324	1,765
Ln(Income, child)-Ln(Income, father) (FD3)	-1,089	1,774
Household characteristics		
Log total expenditure	4,241	0,613
Quarter1	0,298	0,457
Quarter2	0,263	0,440
Quarter3	0,214	0,410
North	0,069	0,254
Yorks/Humerside	0,102	0,302
North West	0,115	0,319
East Midlands	0,079	0,270
West Midlands	0,104	0,305
East Anglia	0,039	0,194
Greater London	0,073	0,261
South East	0,190	0,392
South West	0,078	0,268
Car	0,832	0,374
House	0,489	0,500
Age man	52,071	6,551
Age woman	49,449	5,812
Age child	20,952	4,131
Sex child 1=male	0,577	0,494
Education father	10,425	2,188
Education mother	10,450	2,885
Education child	9,494	4,773
Sample size	2745	

The amounts are in pounds sterling.

Table 2: Estimates of the unconditional demand system

Variable	Food	Alc.	Tobac.	Cloth.	Leisure	Trans.	Serv.	Rest.	P.G.	R.G.
	DISTRIBUTION FACTORS									
FD1	0,011	-0,002	0,002	-0,002	-0,002	-0,002	0,000	0,001	0,001	-0,005
	(3,490)	(1,240)	(1,080)	(0,930)	(0,970)	(1,240)	(0,050)	(1,130)	(0,560)	(2,610)
FD2	0,003	0,000	0,000	0,000	-0,002	-0,001	0,000	0,001	0,000	0,001
	(1,900)	(0,360)	(0,300)	(0,320)	(2,060)	(0,860)	(0,280)	(1,130)	(0,070)	(1,470)
FD3	0,001	0,000	0,002	0,001	0,000	0,000	0,000	0,000	0,001	-0,003
	(0,470)	(0,030)	(2,220)	(0,650)	(0,530)	(0,510)	(0,080)	(0,220)	(0,930)	(3,470)
	PRICE VARIABLES									
Γ -Food	-0,157	-0,003	0,013	-0,033	-0,113	0,137	-0,046	0,074	0,145	0,335
	(1,550)	(0,040)	(0,250)	(0,460)	(2,170)	(3,030)	(1,250)	(1,760)	(2,310)	(5,230)
Γ -Alcohol	-0,011	-0,064	-0,076	-0,025	0,042	0,026	-0,010	-0,013	0,020	-0,106
	(0,300)	(2,760)	(3,900)	(0,920)	(2,150)	(1,510)	(0,750)	(0,830)	(0,860)	(4,400)
Γ-Tobacco	-0,014	0,079	0,014	0,029	0,003	-0,006	-0,151	-0,003	0,055	-0,003
	(0,380)	(3,610)	(0,790)	(1,150)	(0,180)	(0,350)	(11,610)	(0,230)	(2,500)	(0,130)
Γ-Clothing	-0,051	0,017	-0,019	0,011	0,051	-0,041	-0,012	-0,003	-0,018	-0,093
	(0,870)	(0,480)	(0,640)	(0,260)	(1,690)	(1,580)	(0,550)	(0,110)	(0,500)	(2,520)
Γ-Leisure	0,061	0,072	0,034	0,062	-0,096	0,032	0,015	0,036	0,029	0,040
	(2,520)	(4,820)	(2,740)	(3,610)	(7,710)	(2,940)	(1,680)	(3,590)	(1,910)	(2,590)
Γ-Transportation	-0,397	-0,349	0,086	0,260	0,018	0,139	-0,011	0,115	-0,055	0,031
	(3,420)	(4,940)	(1,440)	(3,180)	(0,300)	(2,690)	(0,260)	(2,380)	(0,760)	(0,420)
Γ-Services	-1,690	-0,035	-0,312	0,425	0,233	0,170	0,049	0,015	0,388	0,287
	(4,960)	(0,170)	(1,780)	(1,780)	(1,330)	(1,130)	(0,390)	(0,110)	(1,840)	(1,330)
Γ-Restaurant	0,444	-0,435	-0,123	-0,470	-0,174	-0,192	0,210	-0,048	-0,018	0,076
	(2,070)	(3,330)	(1,110)	(3,110)	(1,580)	(2,020)	(2,700)	(0,540)	(0,140)	(0,560)
Γ-Personal	1,308	0,514	0,246	-0,063	-0,076	-0,140	-0,206	-0,067	-0,085	-0,040
goods	(4,040)	(2,610)	(1,480)	(0,270)	(0,460)	(0,970)	(1,750)	(0,500)	(0,420)	(0,190)
Γ-Recreational	0,553	-0,101	0,151	-0,058	0,000	-0,115	0,277	-0,030	-0,389	-0,497
goods	(2,400)	(0,720)	(1,270)	(0,360)	(0,000)	(1,120)	(3,310)	(0,310)	(2,730)	(3,410)
	PREFERENCE VARIABLES†									
Constant	0,619	-0,197	-0,125	-0,176	0,229	-0,056	-0,025	-0,095	0,069	-0,105
	(10,090)	(5,270)	(3,950)	(4,060)	(7,260)	(2,060)	(1,110)	(3,740)	(1,820)	(2,700)
β Car	0,007	-0,005	-0,006	-0,003	-0,004	0,005	0,002	0,000	0,001	0,006
	(1,160)	(1,400)	(1,880)	(0,630)	(1,080)	(1,620)	(0,650)	(0,090)	(0,340)	(1,500)
β House	0,006	0,003	-0,005	0,007	0,006	0,002	-0,001	-0,001	-0,002	-0,004
	(1,140)	(0,850)	(1,830)	(2,010)	(2,500)	(1,000)	(0,660)	(0,390)	(0,740)	(1,250)
β Constant	-0,146	0,112	0,057	0,066	-0,088	0,019	0,037	0,044	0,007	0,153
	(5,810)	(7,340)	(4,420)	(3,740)	(6,840)	(1,730)	(4,020)	(4,200)	(0,440)	(9,650)
λ	0,002	-0,010	-0,007	-0,005	0,012	-0,002	-0,005	-0,005	-0,002	-0,014
	(0,980)	(7,090)	(5,580)	(2,820)	(10,340)	(2,120)	(5,370)	(4,970)	(1,480)	(9,300)
Test for exogeneity										
Tot. Exp. $\mid t \mid$	4,370	0,760	0,790	2,070	1,690	1,100	0,930	1,190	1,850	3,070
Over-Ident. $\chi^2_{(3)}$	5,401	3,495	3,485	3,089	2,911	1,336	6,668	2,551	1,840	6,745

t-statistics in parentheses.

Table 3: Estimates of the demand system conditional on the share of food

Variable	Alcohol	Tobac.	Cloth.	Leisure	Trans.	Serv.	Rest.	P.G.	L.G.
	CONDITIONING GOOD AND DISTRIBUTION FACTOR								
Food Share	-0,320	-0,046	-0,208	-0,068	-0,124	-0,064	-0,005	-0,226	0,072
	(5,490)	(0,940)	(3,070)	(1,370)	(2,880)	(1,840)	(0,110)	(3,850)	(1,180)
FD2	0,001	0,000	0,000	-0,001	0,000	0,000	0,000	0,000	0,002
	(1,240)	(0,090)	(0,220)	(1,750)	(0,250)	(0,380)	(0,740)	(0,040)	(2,660)
FD3	0,000	0,001	0,001	0,000	0,000	0,000	0,000	0,000	-0,002
	(0,010)	(1,850)	(0,830)	(0,280)	(0,830)	(0,290)	(0,270)	(0,400)	(2,530)
	PRICE VARIABLES								
Γ-Food	-0,042	0,009	-0,060	-0,123	0,121	-0,055	0,075	0,118	0,334
	(0,690)	(0,170)	(0,840)	(2,350)	(2,670)	(1,500)	(1,770)	(1,910)	(5,210)
Γ-Alcohol	-0,071	-0,078	-0,029	0,041	0,023	-0,012	-0,014	0,015	-0,101
	(3,100)	(3,990)	(1,080)	(2,080)	(1,360)	(0,840)	(0,870)	(0,640)	(4,200)
Γ-Tobacco	0,072	0,012	0,025	0,003	-0,007	-0,152	-0,005	0,049	0,003
	(3,370)	(0,650)	(1,020)	(0,150)	(0,460)	(11,860)	(0,320)	(2,250)	(0,150)
Γ-Clothing	-0,003	-0,022	-0,002	0,046	-0,049	-0,015	-0,003	-0,031	-0,087
	(0,100)	(0,720)	(0,060)	(1,530)	(1,880)	(0,710)	(0,110)	(0,880)	(2,360)
Γ-Leisure	0,096	0,037	0,077	-0,091	0,041	0,019	0,036	0,045	0,033
	(6,230)	(2,850)	(4,320)	(6,940)	(3,650)	(2,070)	(3,430)	(2,890)	(2,040)
Γ-Transportation	-0,478	0,065	0,176	-0,009	0,089	-0,037	0,111	-0,148	0,066
	(6,470)	(1,040)	(2,050)	(0,140)	(1,640)	(0,840)	(2,190)	(1,990)	(0,850)
Γ-Services	-0,576	-0,397	0,075	0,121	-0,037	-0,061	0,003	-0,004	0,427
	(2,530)	(2,050)	(0,280)	(0,620)	(0,220)	(0,450)	(0,020)	(0,020)	(1,790)
Γ-Restaurant	-0,291	-0,096	-0,377	-0,147	-0,139	0,241	-0,042	0,092	0,027
	(2,200)	(0,850)	(2,460)	(1,300)	(1,430)	(3,050)	(0,460)	(0,690)	(0,200)
Γ-Personal	0,932	0,312	0,208	0,010	0,019	-0,121	-0,057	0,218	-0,146
goods	(4,440)	(1,740)	(0,850)	(0,060)	(0,130)	(0,970)	(0,400)	(1,030)	(0,670)
Γ-Recreational	0,072	0,174	0,055	0,038	-0,047	0,311	-0,029	-0,269	-0,528
goods	(0,510)	(1,440)	(0,330)	(0,310)	(0,450)	(3,670)	(0,300)	(1,880)	(3,550)
	PREFERENCE VARIABLES†								
Constant	-0,014	-0,090	-0,057	0,264	0,012	0,016	-0,087	0,212	-0,158
	(0,280)	(2,130)	(0,980)	(6,230)	(0,320)	(0,550)	(2,540)	(4,230)	(3,030)
β Car	-0,003	-0,005	-0,001	-0,003	0,005	0,002	0,000	0,004	0,003
	(0,690)	(1,540)	(0,280)	(1,010)	(1,900)	(0,980)	(0,130)	(1,100)	(0,850)
β House	0,007	-0,005	0,006	0,003	0,	-0,001	0,000	0,000	-0,005
	(1,520)	(1,560)	(2,350)	(2,570)	(1,270)	(0,380)	(0,240)	(0,110)	(1,740)
β Constant	0,074	0,052	0,041	-0,097	0,005	0,028	0,043	-0,021	0,157
	(4,530)	(3,690)	(2,130)	(6,900)	(0,380)	(2,860)	(3,830)	(1,290)	(9,160)
λ	-0,010	-0,007	-0,005	0,012	-0,002	-0,004	-0,005	-0,002	-0,013
	(7,230)	(5,580)	(2,830)	(10,320)	(2,170)	(5,310)	(4,960)	(1,450)	(9,060)
Test for exogeneity									
Tot. Exp. $\mid t \mid$	3,280	0,520	2,660	0,310	2,410	0,910	0,460	0,610	0,340
Share Food $\mid t \mid$	4,300	0,280	2,090	1,230	2,450	0,300	0,300	1,850	2,190
Over-Ident. $\chi^2_{(5)}$	10,644	4,918	2,737	7,270	0,732	2,439	5,787	3,551	3,551

t-statistics in parentheses.

Table 4: Estimates of the demand system conditional on the shares of food and leisure

Variable	Alcohol	Tobac.	Cloth.	Leisure	Transp.	Serv.	Rest.	P.G.
	CONDITIONING GOODS AND DISTRIBUTION FACTORS							
Food Share	-0,305	-0,036	-0,208	-0,052	-0,128	-0,059	-0,003	-0,219
	(5,190)	(0,720)	(3,060)	(1,040)	(2,950)	(1,690)	(0,070)	(3,720)
Leisure Share	-0,138	-0,157	0,069	-0,312	0,076	-0,047	0,006	-0,067
	(0,760)	(1,010)	(0,330)	(2,020)	(0,570)	(0,430)	(0,040)	(0,370)
FD3	0,000	0,001	0,001	-0,001	0,001	0,000	0,000	0,000
	(0,100)	(1,300)	(0,970)	(1,340)	(0,970)	(0,370)	(0,120)	(0,270)
	PRICE VARIABLES							
Γ -Food	0,002	0,048	-0,054	-0,050	0,110	-0,040	0,076	0,152
	(0,030)	(0,790)	(0,660)	(0,820)	(2,080)	(0,930)	(1,540)	(2,150)
Γ -Alcohol	-0,084	-0,090	-0,030	0,018	0,026	-0,016	-0,014	0,005
	(3,320)	(4,150)	(1,040)	(0,850)	(1,400)	(1,050)	(0,790)	(0,180)
Γ -Tobacco	0,074	0,014	0,023	0,006	-0,009	-0,152	-0,005	0,048
	(3,430)	(0,730)	(0,930)	(0,320)	(0,550)	(11,790)	(0,320)	(2,240)
Γ -Clothing	-0,014	-0,029	-0,008	0,032	-0,049	-0,019	-0,003	-0,042
	(0,400)	(0,970)	(0,200)	(1,050)	(1,840)	(0,880)	(0,130)	(1,190)
Γ -Leisure	0,098	0,036	0,088	-0,095	0,046	0,020	0,037	0,052
	(6,000)	(2,550)	(4,650)	(6,800)	(3,790)	(2,030)	(3,270)	(3,200)
Γ -Transportation	-0,467	0,078	0,169	0,014	0,082	-0,033	0,111	-0,145
	(6,230)	(1,210)	(1,940)	(0,220)	(1,480)	(0,740)	(2,150)	(1,930)
Γ -Services	-0,507	-0,334	0,058	0,236	-0,065	-0,039	0,005	0,031
	(2,140)	(1,650)	(0,210)	(1,160)	(0,370)	(0,280)	(0,030)	(0,130)
Γ -Restaurant	-0,292	-0,091	-0,382	-0,133	-0,141	0,241	-0,044	0,092
	(2,230)	(0,810)	(2,510)	(1,190)	(1,450)	(3,060)	(0,480)	(0,700)
Γ -Personal	0,913	0,297	0,201	-0,017	0,022	-0,127	-0,058	0,201
goods	(4,380)	(1,660)	(0,830)	(0,100)	(0,140)	(1,020)	(0,400)	(0,960)
Γ -Recreational	-0,007	0,094	0,080	-0,113	-0,011	0,285	-0,029	-0,310
goods	(0,040)	(0,660)	(0,410)	(0,790)	(0,090)	(2,860)	(0,250)	(1,860)
	PREFERENCE VARIABLES†							
Constant	-0,034	-0,087	-0,105	0,282	-0,006	0,010	-0,093	0,176
	(0,640)	(1,880)	(1,680)	(6,100)	(0,150)	(0,310)	(2,480)	(3,260)
β Car	-0,002	-0,005	-0,001	-0,002	0,005	0,002	0,000	0,005
	(0,590)	(1,360)	(0,330)	(0,670)	(1,790)	(1,030)	(0,110)	(1,160)
β House	0,004	-0,005	0,008	0,005	0,003	-0,001	0,000	-0,001
	(1,260)	(1,790)	(2,340)	(1,940)	(1,360)	(0,500)	(0,210)	(0,250)
β Constant	0,094	0,064	0,056	-0,076	0,006	0,035	0,045	0,000
	(5,360)	(4,230)	(2,740)	(5,060)	(0,450)	(3,290)	(3,680)	(0,020)
λ	-0,012	-0,008	-0,006	0,011	-0,002	-0,005	-0,005	-0,004
	(8,150)	(6,120)	(3,650)	(8,780)	(2,310)	(5,760)	(4,920)	(2,730)
Test for exogeneity								
Tot. Exp. $\mid t \mid$	3,130	0,320	2,890	0,670	2,560	0,880	0,470	0,410
Share Food $\mid t \mid$	3,890	0,000	1,950	0,810	2,470	0,100	0,360	1,550
Share Leis. $\mid t \mid$	0,090	0,660	1,010	1,500	0,810	0,060	0,160	0,490
Over-Ident. $\chi^2_{(5)}$	2,966	2,732	6,562	0,639	1,959	3,805	3,900	3,900

t-statistics in parentheses.