Joint Labour Participation*

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Abstract

For a household with preferences depending on consumption and the leisures of two potential workers, we examine the joint labour participation decisions. Generally there are regions of wages with two, one or no workers. But if cross wage effects are strong, there will never be two workers.

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Typically there is heterogeneity in the labour force participation pattern of households with two adults. For example in the 2003-4 UK Expenditure & Food Survey for about 3000 two adult households with household reference person of working age, 11% had no workers, 27% had one worker and 62% had two workers. So most households are making decisions on joint labour supply. The theory of this is still not fully advanced, most studies of joint participation use simulation methods or discrete hours and then grid search to determine participation. Hausman and Ruud (1984) estimate joint labour supply solving numerically for virtual incomes and wages in an empirical application with particular preferences. Our aim is to characterise these virtual wages theoretically

In this paper we use a household utility approach to analyse the joint labour supply decisions of couples with flexible hours of work and no involuntary unemployment. We find that the wage space divides into regions with no, one or two workers in a way which depends on the relative strengths of own and cross wage effects and of income effects. For example, if leisures are not Giffen and the two leisures are substitutes, then so long as the own wage effects are stronger than the cross wage effects, the pattern of work is as expected: with low wages for both individuals neither work, with high wages for both, both work and with a high wage for the first person but a low wage for the second, only the first works. But surprisingly, if cross wage effects are stronger than own wage effects (and the other conditions apply), there are no wages at which both individuals will work. We can also derive comparative static implications, for example of the effects of household non-labour income on the work pattern.

1 The Analytical Framework

A household has two adults and preferences defined by the increasing, strictly quasiconcave C^2 utility $u(x, l_1, l_2)$ where x is household consumption and l_i leisure of individual i. Each adult i has a time endowment of T_i that can be used for market work at a constant hourly wage of w_i or for leisure. The household has non-labour income of y. It follows that the household takes decisions to

$$\max u(x, l_1, l_2) \tag{1}$$

$$\text{s.t.} \quad x + w_1 l_1 + w_2 l_2 = y + w_1 T_1 + w_2 T_2 \tag{2}$$

$$0 \leq l_i \leq T_i \tag{3}$$

Since the problem satisfies the convexity conditions and is smooth, the solution solves the system:

$$\frac{\partial u}{\partial x} = \lambda \tag{4}$$

$$\frac{\partial u}{\partial l_i} \geq \lambda w_i, \frac{\partial u}{\partial l_i}(T_i - l_i) = \lambda w_i (T_i - l_i), T_i \geq l_i \tag{5}$$

$$x + w_1 l_1 + w_2 l_2 = y + w_1 T_1 + w_2 T_2 \tag{6}$$

1.1 Participation Decisions

The possible regimes are:

(1) Neither works: $l_1 = T_1, l_2 = T_2; x = y$

From (4)-(6) this is optimal if:

$$\frac{\partial u(y, T_1, T_2)}{\partial l_i} / \frac{\partial u(y, T_1, T_2)}{\partial x} \geq w_i, i = 1, 2 \tag{7}$$

So no worker reservation wages are given by:

$$\frac{\partial u(y, T_1, T_2)}{\partial l_i} / \frac{\partial u(y, T_1, T_2)}{\partial x} = w_i^{2R}(y, T_1, T_2), i = 1, 2 \tag{8}$$

Neither individual works if $w_i \leq w_i^R$ for $i = 1, 2$[1].

(2) Only 1 works: $l_1 < T_1, l_2 = T_2; x = y + w_1(T_1 - l_1)$

From (4)-(6) this is optimal iff:

$$\begin{aligned} \frac{\partial u(y,T_1,T_2)}{\partial l_1} \Big/ \frac{\partial u(y,T_1,T_2)}{\partial x} &< w_1 \\ \frac{\partial u(x,l_1,T_2)}{\partial l_1} \Big/ \frac{\partial u(x,l_1,T_2)}{\partial x} &= w_1 \\ \frac{\partial u(x,l_1,T)_2}{\partial l_2} \Big/ \frac{\partial u(x,l_1,T_2)}{\partial x} &> w_2 \end{aligned} \tag{9}$$

Let $\widetilde{L}_1(w_1, y)$ and $W_2(w_1, y)$ solve:

$$\frac{\partial u(x,\widetilde{L}_1(w_1,y),T_2)}{\partial l_1} \Big/ \frac{\partial u(x,\widetilde{L}_1(w_1,y),T_2)}{\partial x} = w_1 \tag{10}$$

$$\frac{\partial u(x,\widetilde{L}_1(w_1,y),T_2)}{\partial l_2} \Big/ \frac{\partial u(x,\widetilde{L}_1(w_1,y),T_2)}{\partial x} = W_2(w_1,y) \tag{11}$$

where w_1 satisfies (9).

Notice that:

- $$\frac{\partial u(y,l_1,T_2)}{\partial l_1} \Big/ \frac{\partial u(y,l_1,T_2)}{\partial x} > \frac{\partial u(y,T_1,T_2)}{\partial l_1} \Big/ \frac{\partial u(y,T_1,T_2)}{\partial x} = w_1^{2R} \tag{12}$$

  for $l_1 < T_1$ so for 1 to work but 2 not to work requires $w_1 > w_1^{2R}$.

- we may have

$$\begin{aligned} w_2^{2R} &= \frac{\partial u(x,T_1,T_2)}{\partial l_2} \Big/ \frac{\partial u(x,T_1,T_2)}{\partial x} \\ &\gtrless \frac{\partial u(x,L_1(w_1,y),T_2)}{\partial l_2} \Big/ \frac{\partial u(x,L_1(w_1,y),T_2)}{\partial x} = W_2(w_1,y) \end{aligned}$$

  That is when 1 works, the wage at which 2 drops out of work may be above or below that at which 2 drops out of the labour market when 1

[1] Equivalently we can derive these reservation wages from solving the equations $L_i(w_1, w_2, y) = T, i = 1, 2$.

does not work, it depends on how the mrs between l_2 and x varies with l_1.

- Since $\widetilde{L}_1(w_1, y)$ is independent of w_2, if $w_2 < W_2(w_1, y)$ then 2 does not work.

(3) Only 2 works: $l_2 < T_2, l_1 = T_1; x = y + w_2(T_2 - l_2)$

Analogously $\widetilde{L}_2(w_2, y)$ and $W_1(w_2, y)$ solve:

$$\frac{\partial u(x, T_1, \widetilde{L}_2(w_2, y))}{\partial l_2} / \frac{\partial u(x, T_1, \widetilde{L}_2(w_2, y))}{\partial x} = w_2$$
$$\frac{\partial u(x, T_1, \widetilde{L}_2(w_2, y))}{\partial l_1} / \frac{\partial u(x, T_1, \widetilde{L}_2(w_2, y))}{\partial x} = W_1(w_2, y)$$

(4) Both work: $l_1 < T_1, l_2 < T_2, x = y + w_1(T_1 - l_1) + w_2(T_2 - l_2)$

Here $L_1(w_1, w_2, y), L_2(w_1, w_2, y)$ solve

$$\frac{\partial u(x, L_1(w_1, w_2, y), L_2(w_1, w_2, y))}{\partial l_2} / \frac{\partial u(x, L_1(w_1, w_2, y), L_2(w_1, w_2, y))}{\partial x} = w_2$$
$$\frac{\partial u(x, L_1(w_1, w_2, y), L_2(w_1, w_2, y))}{\partial l_1} / \frac{\partial u(x, L_1(w_1, w_2, y), L_2(w_1, w_2, y))}{\partial x} = w_1$$

This is optimal if:

$$w_1^{2R} < W_1(w_2, y) < w_1 \tag{13}$$

$$w_2^{2R} < W_2(w_1, y) < w_2 \tag{14}$$

1.2 The Wage Space

The functions w_i^{2R} and W_i determine participation. Note that if $w_1 \leq w_1^{2R}$ then $w_2^{2R} = W_2(y, w_1)$. Hence $W_1(.)$ and $W_2(.)$ must meet at the point (w_1^{2R}, w_2^{2R}).

We can also define, say, $W_1(w_2, y)$ by $L_1(W_1(w_2, y), w_2, y) = T_1$, from which we get:

$$\frac{\partial W_1}{\partial w_2} = -\frac{\partial L_1}{\partial w_2} / \frac{\partial L_1}{\partial w_1} \tag{15}$$

Assume that leisure is not Giffen, so that $\partial L_1/\partial w_1 < 0$. Then $W_i()$ is increasing in w_j when the lesiures are gross substitutes ($\partial L_i/\partial w_j > 0$) and decreasing with gross complements ($\partial L_i/\partial w_j < 0$)). Under substitutability the wage space divides either as in Fig 1 or Fig 2. In fact the region between $W_1()$ and $W_2()$ in Fig 2 can be further divided.

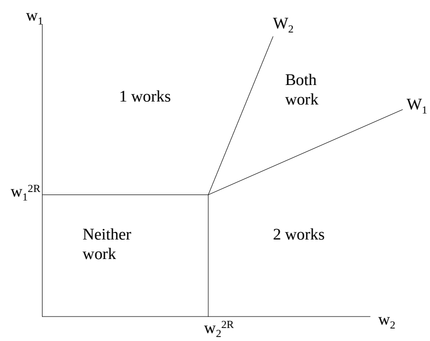

Figure 1

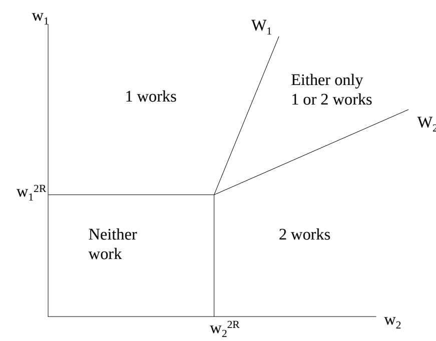

Figure 2

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In Figure 2 take any point between the W_1 and W_2 loci. With these wages it cannot be that both individuals work. For each individual i it is true that $w_i < W_i$ so that, conditional on individual j working, the best household choice is for i not to work. Similarly since both wages are above w_i^{2R} it cannot be optimal for neither individual to work. Thus it must be that just one individual works. To save notation define utility after elimination of the budget constraint as

$$v(w_1, w_2, l_1, l_2, y) = u(y + w_1(T_1 - l_1) + w_2(T_2 - L_2), l_1, l_2)$$

Along the line AB in Fig 3 the utility $v(w_1, w_2, L_1(w_1, y, T_2), T_2, y)$ is constant but the utility $v(w_1, w_2, T_1, L_2(w_2, y, T_1))$ increases with w_2. We also know that at B' the global maximum has $l_1 = T_1$ and $l_2 < T_2$. Thus at B'

$$v(w_1, W_2(w_1, y)+\varepsilon, T_1, L_2(W_2(w_1, y)+\varepsilon, y) > v(w_1, W_2(w_1, y)+\varepsilon, L_1(w_1, y, T_2), y)$$

Hence by continuity of utility and leisure demand at B

$$v(w_1, W_2(w_1, y), T_1, L_2(W_2(w_1, y), y) \geq v(w_1, W_2(w_1, y), L_1(w_1, y, T_2), y)$$

Since the global maximum is unique, we can preserve the strict inequality. Thus at and near B it is better for only 2 to work. Similarly at and near A it is better for only 1 to work. Hence there must be a locus dividing the area between W_1 and W_2 determining which individual will work.

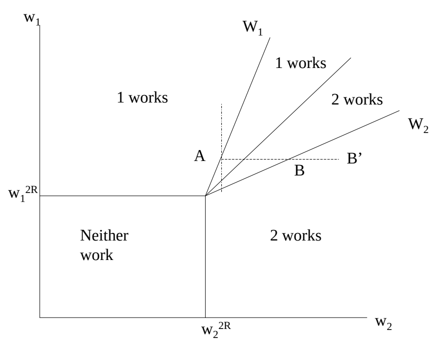

Figure 3

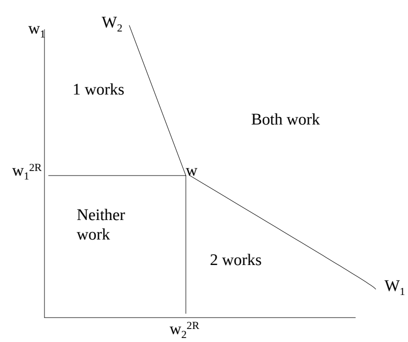

Figure 4

What property of preferences distinguishes Figs 1,2? In Figure 1 $\partial W_2/\partial w_1 \partial W_1/\partial w_2 < 1$ whereas in Figure 2 $\partial W_2/\partial w_1 \partial W_1/\partial w_2 > 1$. Since:

$$\frac{\partial W_i}{\partial w_j} = -\frac{\partial L_i}{\partial w_j} / \frac{\partial L_i}{\partial w_i} \tag{16}$$

in Figure 1 we have:

$$\frac{\partial L_1}{\partial w_1}\frac{\partial L_2}{\partial w_2} > \frac{\partial L_1}{\partial w_2}\frac{\partial L_2}{\partial w_1} \tag{17}$$

whereas in Figs 3-4 we have:

$$\frac{\partial L_1}{\partial w_1}\frac{\partial L_2}{\partial w_2} < \frac{\partial L_1}{\partial w_2}\frac{\partial L_2}{\partial w_1} \tag{18}$$

Exploring this further, Slutsky's equation implies:

$$\frac{\partial L_i}{\partial w_j} = s_{ij} + \frac{\partial L_i}{\partial y} l_j, i = 1, 2 \tag{19}$$

where s_{ij} is the substitution effect, so:

$$\frac{\partial L_2}{\partial w_1}\frac{\partial L_1}{\partial w_2} - \frac{\partial L_2}{\partial w_2}\frac{\partial L_1}{\partial w_1} = s_{12}s_{21} - s_{11}s_{22} + \frac{\partial L_1}{\partial y}(s_{21}l_2 - s_{22}l_1) + \frac{\partial L_2}{\partial y}(s_{12}l_1 - s_{11}l_2) \tag{20}$$

The first term is negative, if leisures are normal the last two terms are positive when the two leisures are net substitutes. Hence with strong income effects on leisure and strong substitutability between leisures the whole expression tends to be positive. So households like this will never have two workers. These sorts of effects could arise if there are single person tasks in home production, or with high priority but competing and exclusive leisure activities e.g. the household has just one ticket to the World Cup game. An extreme parametric example would be $u(x, l_1, l_2) = x^\alpha + (l_1 + l_2)^\alpha, 0 < \alpha < 1$.

In Figure 4 the two leisures are complements. Here there is a wider range of wages for which the participation decisions of the two individuals are identical, either both or neither working. Note that it is not possible for the loci $W_1(.)$ and $W_2(.)$ to be reversed in Fig 4 (this would require only 2 to work at a point with $w_1 > w_1^{2R}, w_2 < w_2^{2R}$). The example $u(x, l_1, l_2) = x^\alpha + (l_1 l_2)^\alpha, 0 < \alpha < 1$ can generate both Fig 1 and Fig 4 for suitable α, T_i and y.

2 Conclusion

With few exceptions, existing approaches to analysing hours of work and participation of couples either use reduced form bivariate probits (Blundell and MaCurdy,1999) or analyse labour supply of one party conditional on the labour supply of the second person (Blundell at al, 1998) or use a discrete choice model with a restricted finite set of hours (van Soest, 1995). The reduced form approach cannot derive, impose or test for theoretical restrictions. Conditioning on the labour supply of the second person raises issues of estimation strategy arising from endogeneity and also of course cannot explain the joint household decision over who works. In the collective approach Blundell, Chiappori and Magnac (1998) use the conditioning approach but, as Donni (2003) has shown, in this approach, if the household "welfare function" shifts with current wages or other budget constraint variables via the sharing rule then there is no simple idea of a reservation wage frontier. Similarly the discrete choice approach usually leads to a conditional logit model from which it is difficult to infer the reservation wage functions. In a household utility model with two potential workers we have shown that if leisures are normal and own wage effects dominate cross wage effects, the household will have no workers if both wage rates are low, one worker if one wage is high and the other low, two workers if both wages are high. But if cross wage effects dominate own wage effects (which is likely with strong income effects and strongly substitutable leisures), then there will never be two workers.

There are empirical and policy implications. For example, with unobservable individual heterogeneity the appropriate estimation strategy for household labour supplies might use a switching regime approach in which the probability of a household having a particular participation pattern is the chance that the stochastic reservation wages are bounded in particular ways by market wages. In policy concerns, if a household is of the type shown in Figure 2 then reducing the gender pay gap will not encourage participation.

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